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AlphaZ & the Polyhedral Equational Model

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Polyhedral Compilation

- The Polyhedral Model
  - Established approach for automatic parallelization
  - Based on mathematical formalism
- Many tools and compilers:
  - PIPS, PLuTo, MMAlpha, Par4All, RStream, XLF/XLC GRAPHITE (gcc), Polly (LLVM), …
  - and AlphaZ

Design Space (a subset)

- Space-Time + Tiling: schedule + parallel loops
  - Primary focus of existing tools
- Memory Allocation
  - Most tools do not modify the original allocation
  - Complex interaction with space time
- Higher-level Optimizations
  - Detection/parallelization of reductions & scans
  - Simplifying Reductions (complexity reduction)
  - Equational Programming

AlphaZ

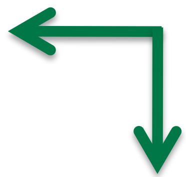
- Tool for Exploration
  - Provides a collection of analyses, transformations, and code generators
  - Unique Features
    - Memory Allocation
    - Reductions
- Can be used as a push-button system (e.g., Parallelization à la PLuTo is possible) but not our current focus
  - [caveat: a push button MPI code generator is now available]

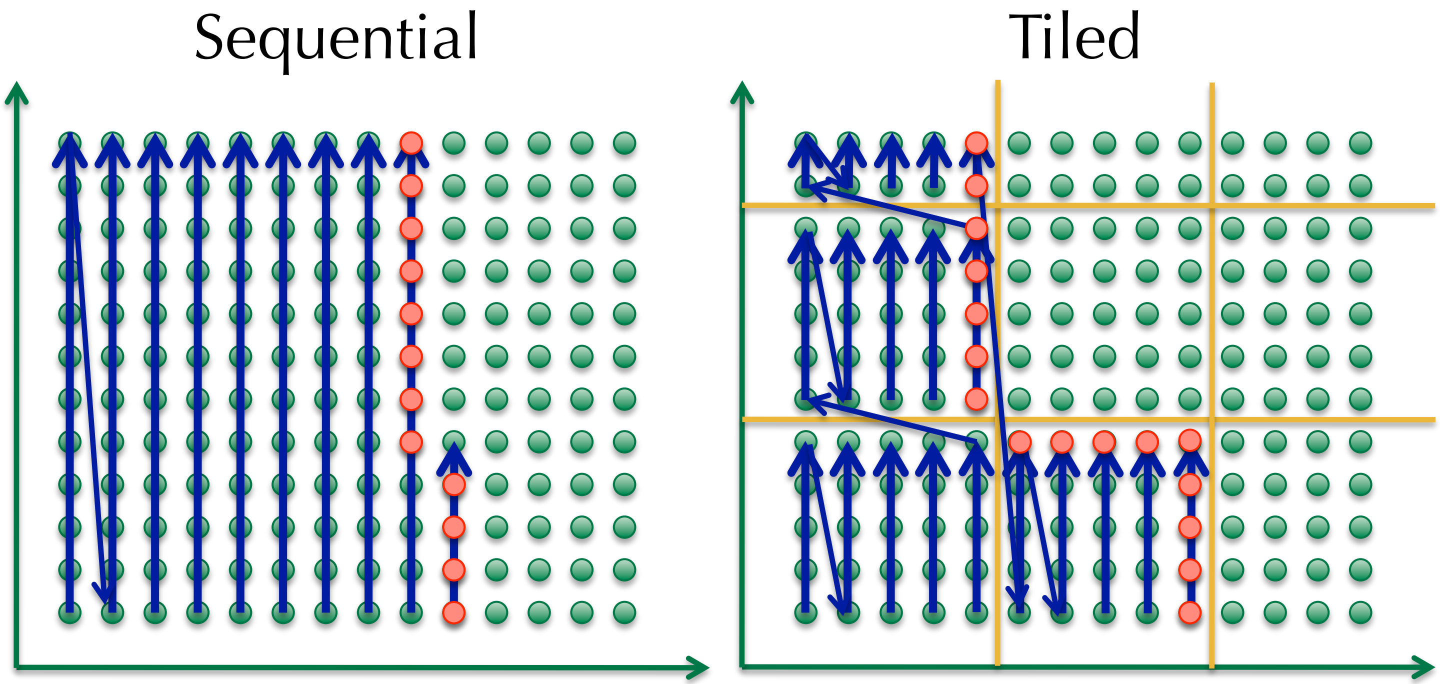
Two Examples

- adi.c from PolyBench
  - Re-considering memory allocation allows the program to be fully tiled
  - Outperforms PLuTo that only tiles inner loops
- LU Decomposition (illustration)
  - Deriving an equational program from first principles

Focus on Memory

- Tiling requires more memory
- e.g., Smith-Waterman dependence





ADI-like Computation

- Updates 2D grid with outer time loop
- PLuTo only tiles inner two dimensions
  - Due to a memory based dependence
  - With an extra **scalar**, all three dimensions can be tiled
- PolyBench implementation has a bug
  - It does not correctly implement ADI
  - All dimensions of a correct ADI program cannot be tiled

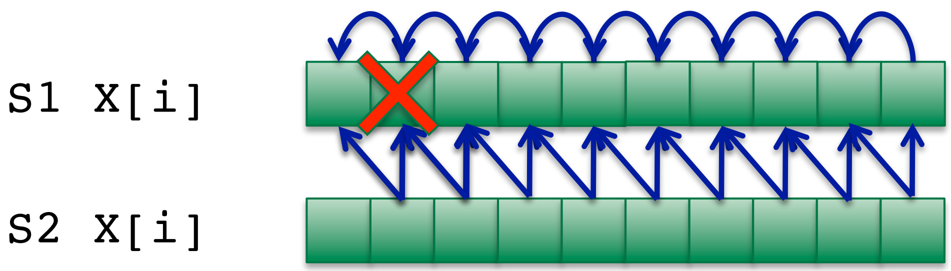
adi.c: Original Allocation

```
for (t=0; t < tsteps; t++) {
   for (i = 0; i < n; i++)
      for (j = 0; j < n; j++)
         X[i][j] = foo(X[i][j], X[i][j-1], …)
   …
   for (i = 0; i < n; i++)
      for (j = n-1; j >= 1; j--)
         X[i][j] = bar(X[i][j], X[i][j-1], …)
   …
}
```

- Not tilable because of the reverse loop
  - Memory based dependence: (i,j -> i,j+1)
  - Requires all dependences to be non-negative

adi.c: Original Allocation

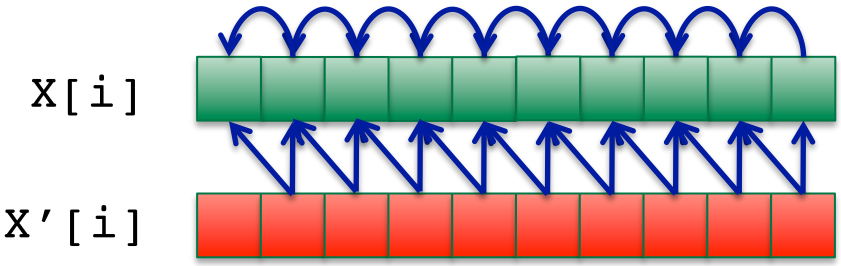
```
        for (j = 0; j < n; j++)
S1:         X[i][j] = foo(X[i][j], X[i][j-1], …)
    …

        for (j = n-1; j >= 1; j--)
S2:         X[i][j] = bar(X[i][j], X[i][j-1], …)
    …
```



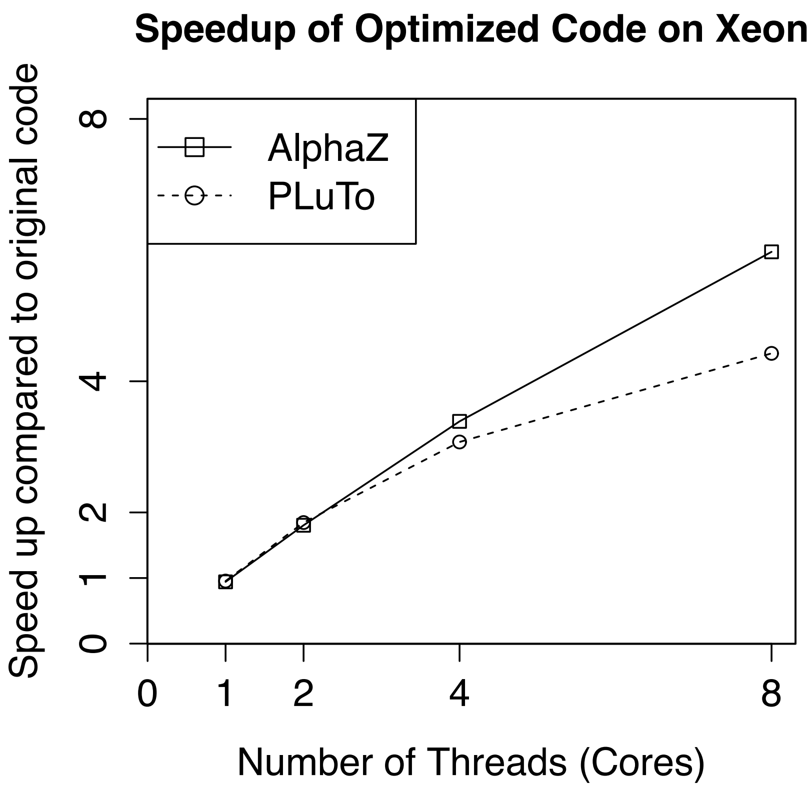
adi.c: With Extra Memory

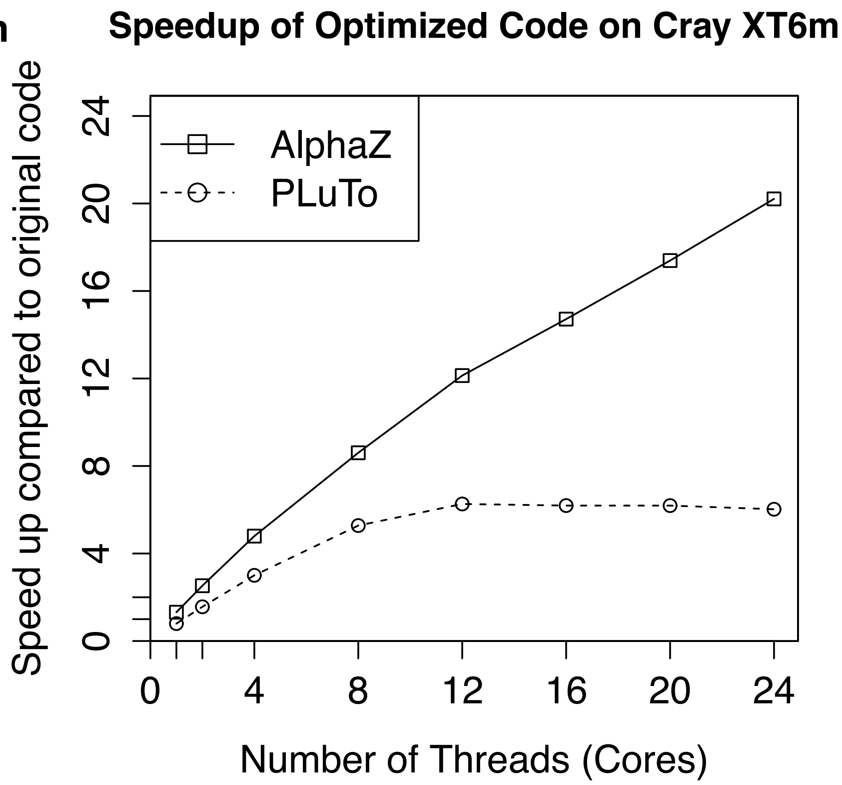
- Once the two loops are fused:
  - Value of X only needs to be preserved for one iteration of j
  - We don’t need a full array X’, just a scalar

```
    for (j = 0; j < n; j++)
S1:     X[i][j] = foo(X[i][j], X[i][j-1], …)
    …
    for (j = 1; j < n; j++)
S2:     X’[i][j] = bar(X[i][j], X[i][j-1], …)
    …
```



adi.c: Performance





- PLuTo does not scale because the outer loop is not tiled

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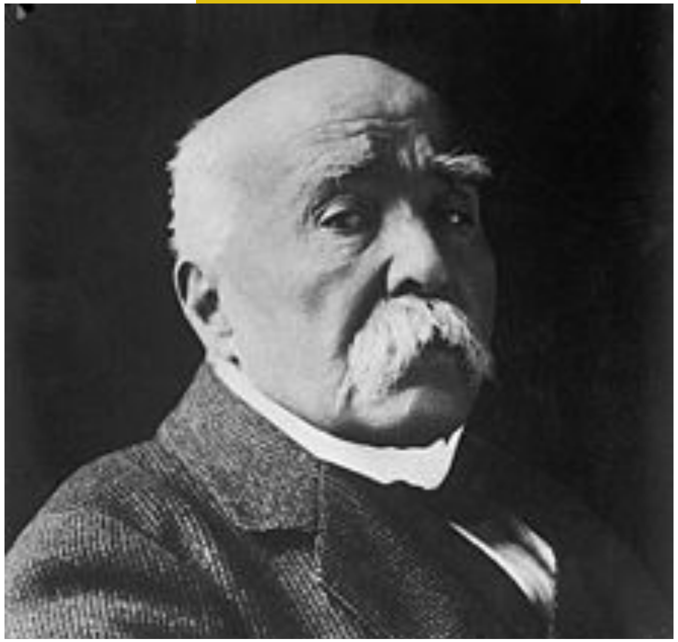
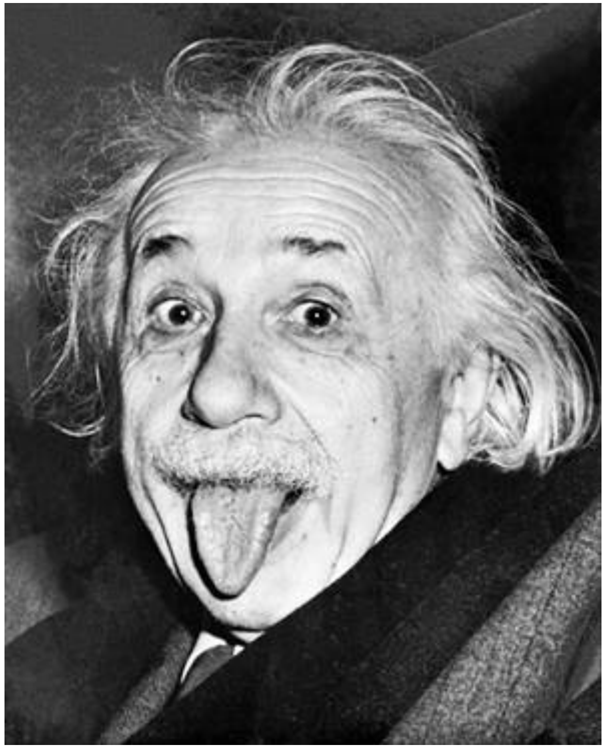
Moral

- War is too serious a matter to entrust to military men.
  - Georges Clemenceau, early 20th century French PM



- Memory is too serious to entrust to programmers

Equational Programming: $E=mc^2$



LU Decomposition (derivation)

$$\boldsymbol{A_{i,j}} = \sum_{k=1}^{\min(i,j)} L_{i,k}U_{k,j}$$

$$\boldsymbol{A_{i,j}} = \begin{cases} \boldsymbol{i \le j} & \sum_{k=1}^{i} L_{i,k}U_{k,j} \qquad U_{i,j} + \sum_{k=1}^{i-1} L_{i,k}U_{k,j} \\ \boldsymbol{i > j} & \sum_{k=1}^{j} L_{i,k}U_{k,j} \qquad L_{i,j}U_{j,j} + \sum_{k=1}^{j-1} L_{i,k}U_{k,j} \end{cases}$$

LU Decomposition (derivation)

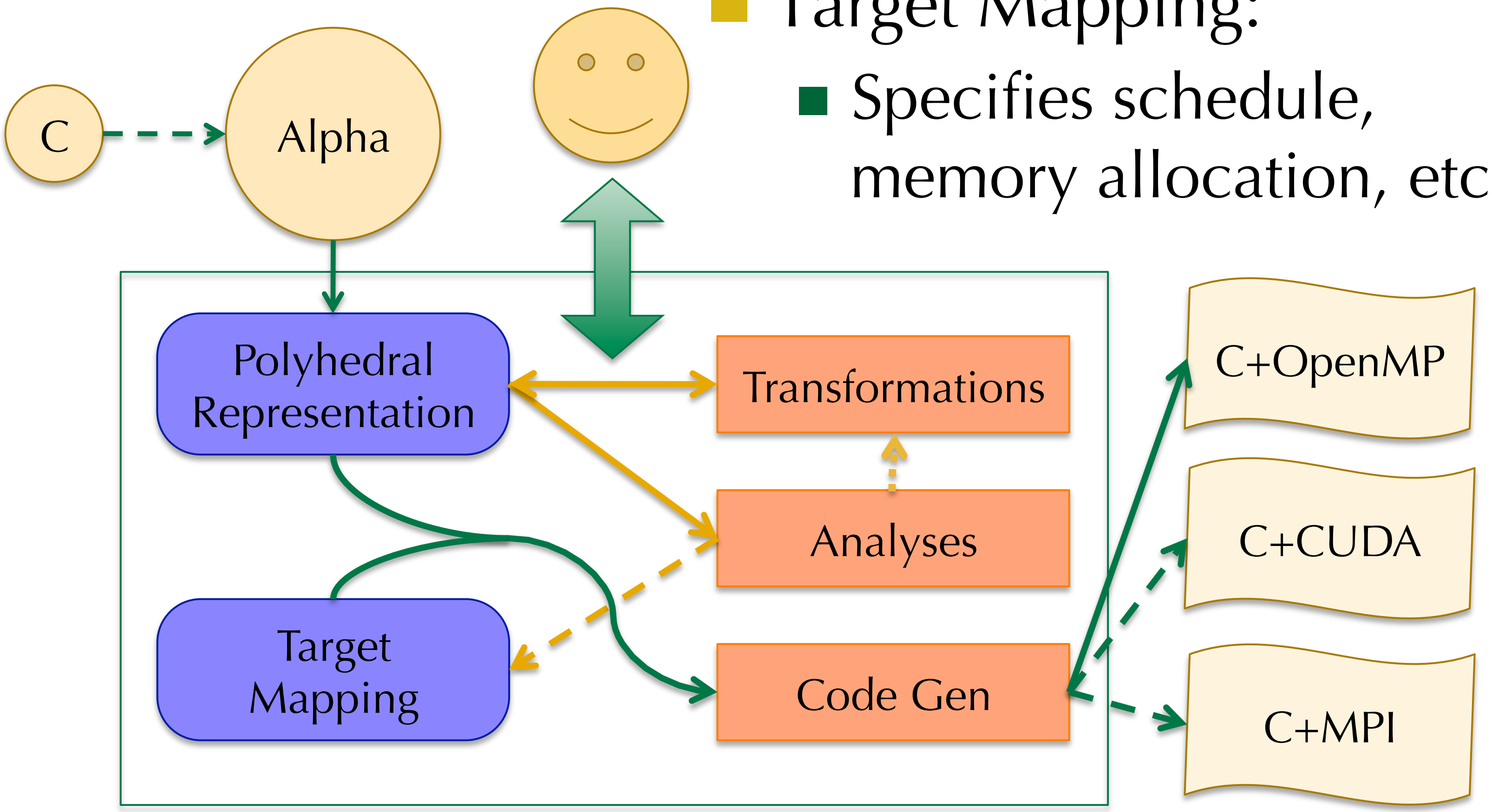
$$U_{i,j} = A_{i,j} - \sum_{k=1}^{i-1} L_{i,k} U_{k,j}$$

$$L_{i,j} = \left(A_{i,j} - \sum_{k=1}^{j-1} L_{i,k} U_{k,j} \right) / U_{j,j}$$

This is the Alpha program

```
affine LUD {N|1<N}
input
  float A {i,j | 0<(i,j)<=N}
output
  float L {i,j | 0<j<i<=N}
  float U {i,j | 0<i<=j<=N}
let
   L[i,j] =  A[i,j] - reduce(+, [k] L[i,k]*U[k,j])
   U[i,j] = (A[i,j] - reduce(+, [k] L[i,k]*U[k,j]))/U[j,j]
}
```

AlphaZ System Overview



- Target Mapping:
  - Specifies schedule, memory allocation, etc.

Human-in-the-Loop

- Automatic parallelization—"holy grail" goal
  - Current automatic tools are restrictive
    - A strategy that works well is "hard-coded"
    - difficult to pass domain specific knowledge
- Human-in-the-Loop
  - Provide full control to the user
    - Help finding new "good" strategies
    - Guide the transformation with domain specific knowledge

Conclusions

- There are more strategies worth exploring
  - some may currently be difficult to automate
- Two examples
  - adi.c: memory
  - Deriving LU decopostition (first principles)
- AlphaZ: Tool for trying out new ideas: see
  - https://www.cs.colostate.edu/AlphaZ/wiki
  - http://www.cs.colostate.edu/TechReports
    - 12-101 [AlphaZ details] & others

Acknowledgements

- AlphaZ Developers/Users
  - Members of MÉLANGE at CSU
  - Members of CAIRN at IRISA, Rennes
  - Dave Wonnacott and students, Haverford University

Key: Simplifying Reductions

- Simplifying Reductions [POPL 2006]
  - Finds “hidden scans” in reductions
  - Rare case: compiler can reduce complexity
- Main idea:

$$X[i] = \sum_{k=0}^{i} A[i] \qquad O(n^2)$$

  - becomes

$$X[i] = \begin{cases} i = 0 : A[i] \\ i > 0 : X[i-1] + A[i] \end{cases} \qquad O(n)$$