Advanced Shortest Paths Algorithms on a Massively-Multithreaded Architecture

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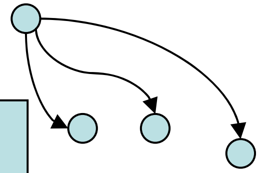
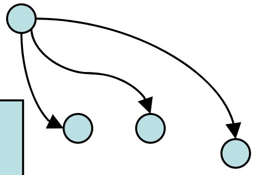


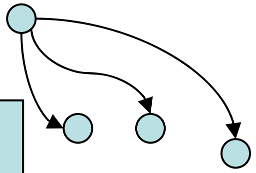
Motivation

- Shortest path are at the core of several real-world graph problems, such as centrality metrics.
- Other Multithreaded SSSP algorithms perform more than optimal work.
- Support for simultaneous SSSP queries.



Our Contributions

- Experimental study of parallel version of Thorup’s algorithm for the single source shortest path problem.
- Memory-efficient mechanism for handling simultaneous shortest path queries.

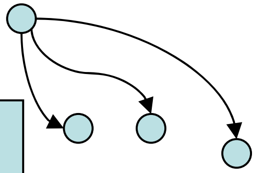


Past Work on Parallel SSSP

- Relaxed heaps [DGST88], [BTZ98].
- Randomization [UY90].
- Delta-Stepping [MS03].
- Distributed memory implementations based on graph partitioning.

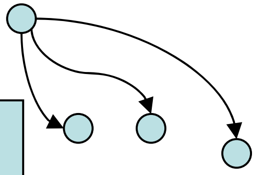
MTA Δ-Stepping [MBBC07]

- Synthetic directed scale-free graph of 100 million vertices and 1 billion edges takes 9.73 seconds.
- Relative speedup of 31 on 40 processors of Cray MTA-2.



Parallel Shortest Path

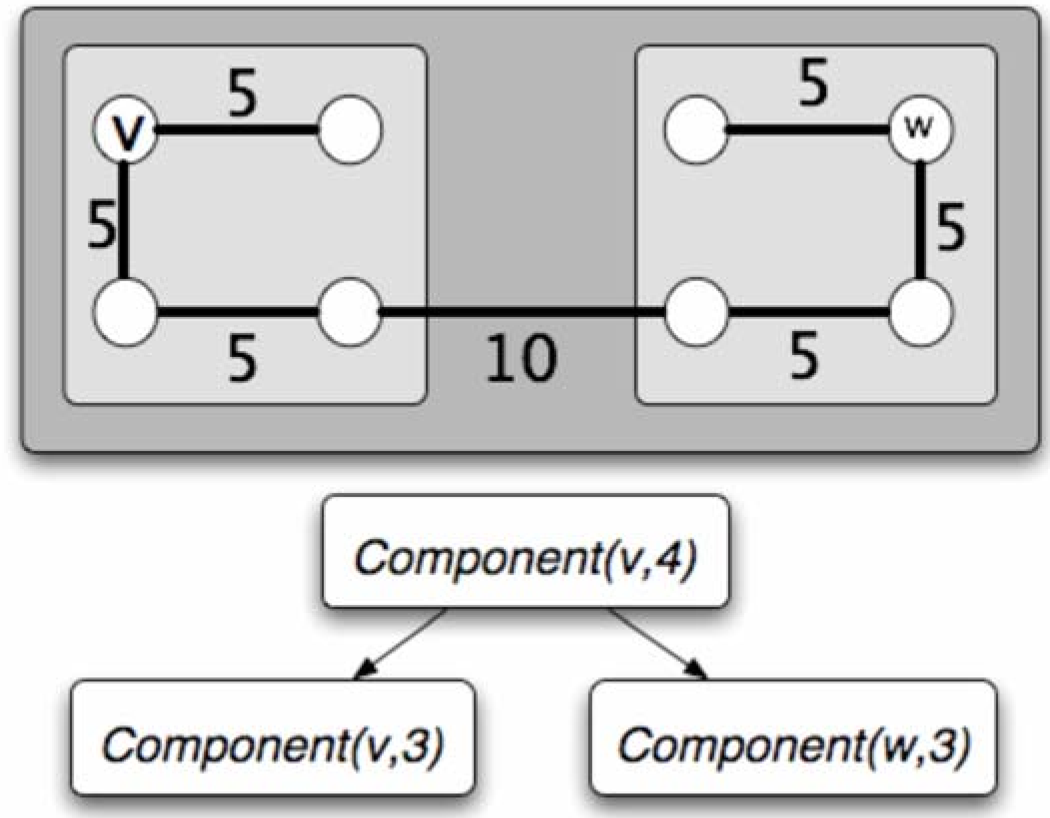
- Problem: Dijkstra-based algorithms inherently serial.
- Solutions: Randomization, heuristics, bucketing (Δ-stepping).

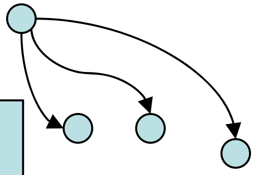


Thorup's algorithm

- Undirected, integer weight edges
- Preprocess to build a tree structure called the *component hierarchy*.
- Scan edges out of source.
- Traverse the component hierarchy, which computes SSSP from source.
- O(n+m) time with n vertices and m edges.

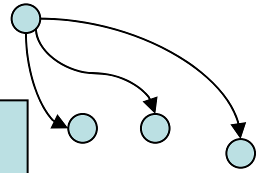
Component Hierarchy





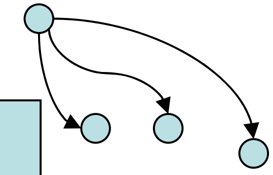
Traversing Component Hierarchy

- Components are *visited* recursively.
  - Determine which children to visit, and visit them recursively.
- If component represents a single vertex, then scan edges out of that vertex.



Component Bucketing

- Each *component* maintains the minimum distance value, *min-D*, for all unsettled vertices in its component.
- A *component* buckets its child *c* based upon *c*'s *min-D*.

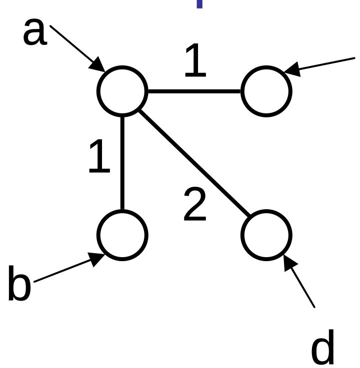


Component Hierarchy in parallel.

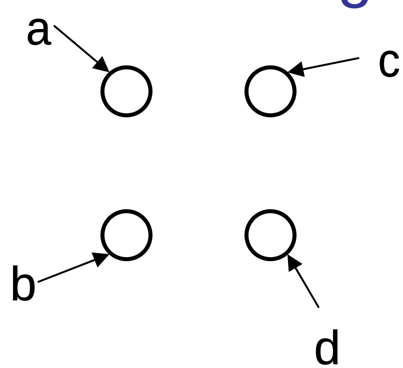
- log(max weight) phases.
- Initially, graph contains only the vertices.
- At phase *i* add edges with weight $<2^i$, collapse connected components into a node in the component hierarchy.

Example Component Hierarchy

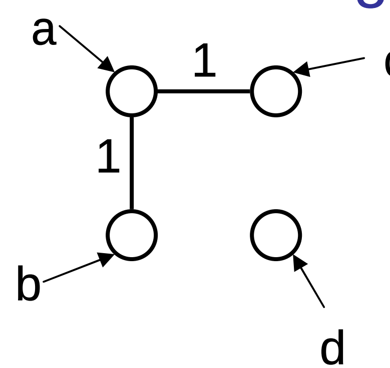
Graph *G*



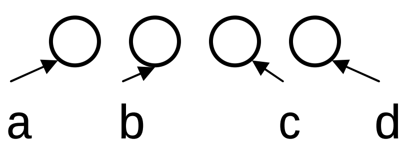
G without edges



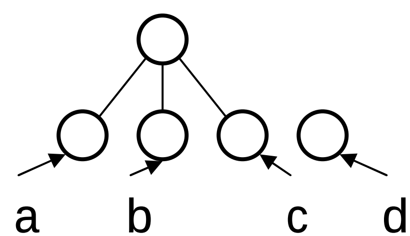
G with w=1 edges



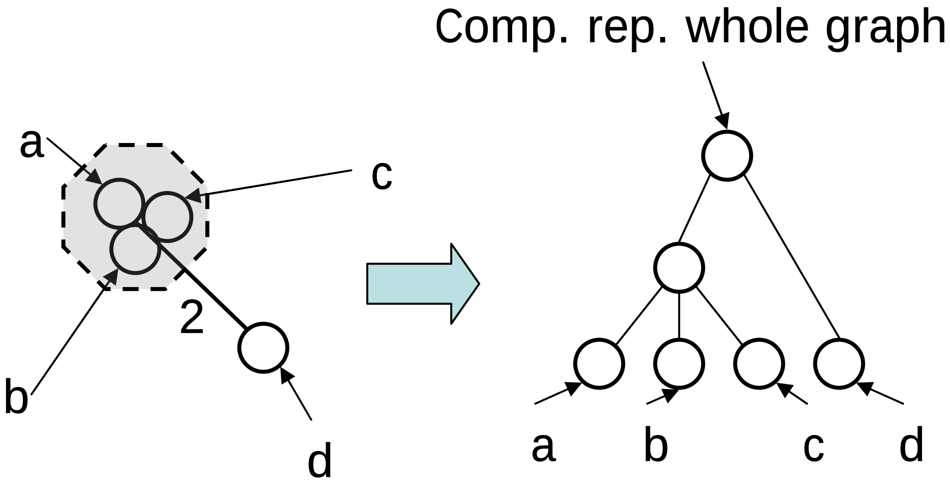
Leaves↓ of CH

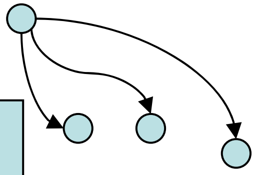


First level↓ of CH



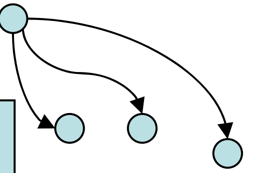
Example (cont.)





Traversing the Component Hierarchy in parallel

- Visiting a component
  - Discover which components are in lowest level bucket in parallel.
- Visiting a leaf component
  - Scan edges in parallel, update *min-D* values.



Implementation details

- The number of children a component has varies from two to several thousand.
- Based upon the number of iterations, we either perform the following loop on all processors, a single processor, or in serial.

```
int index=0;
#pragma mta assert nodep
for (int i=0; i<numChildren; i++) {
    CHNode *c = children_store[i];
    if (bucketOf[c->id] == thisBucket) {
        toVisit[index++] = child->id;
    }
}
```

Cray MTA-2 (XMT)

- Tolerates latency by multi-threading
  - hardware support for 128 threads on each processor
  - Globally hashed address space
  - No data cache
  - Single cycle context switch
  - Multiple outstanding memory requests
- Support for fine-grained, word-level synchronization
  - Full/empty bit associated with every memory word
- Flexibly supports dynamic load balancing
- Clock frequency: 220 MHz, largest machine: 40 processors

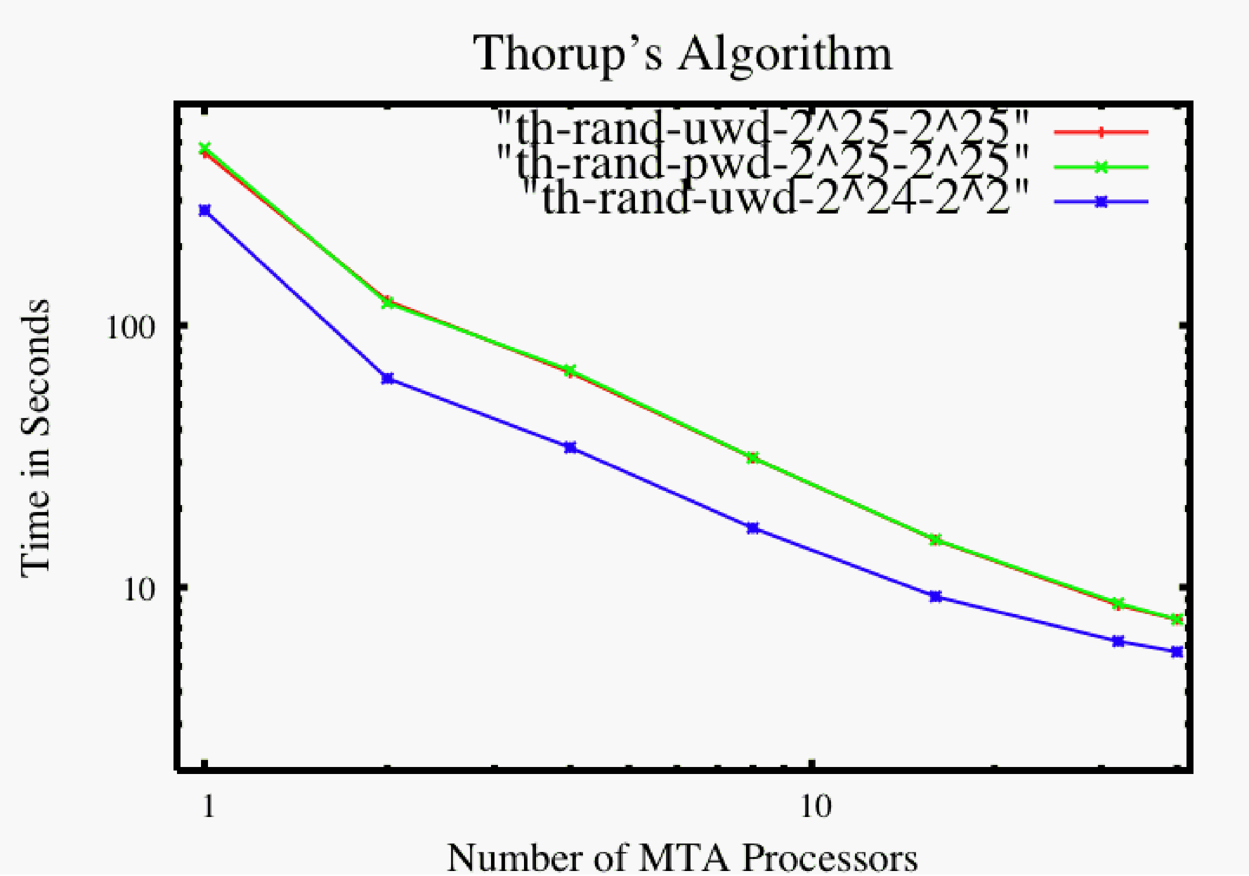
Experimental Setup

- Compared to DIMACS Reference solver for random graphs on a sequential compile on a Linux workstation.
- Compared to Delta-Stepping on random and scale free graphs on the Cray MTA-2.

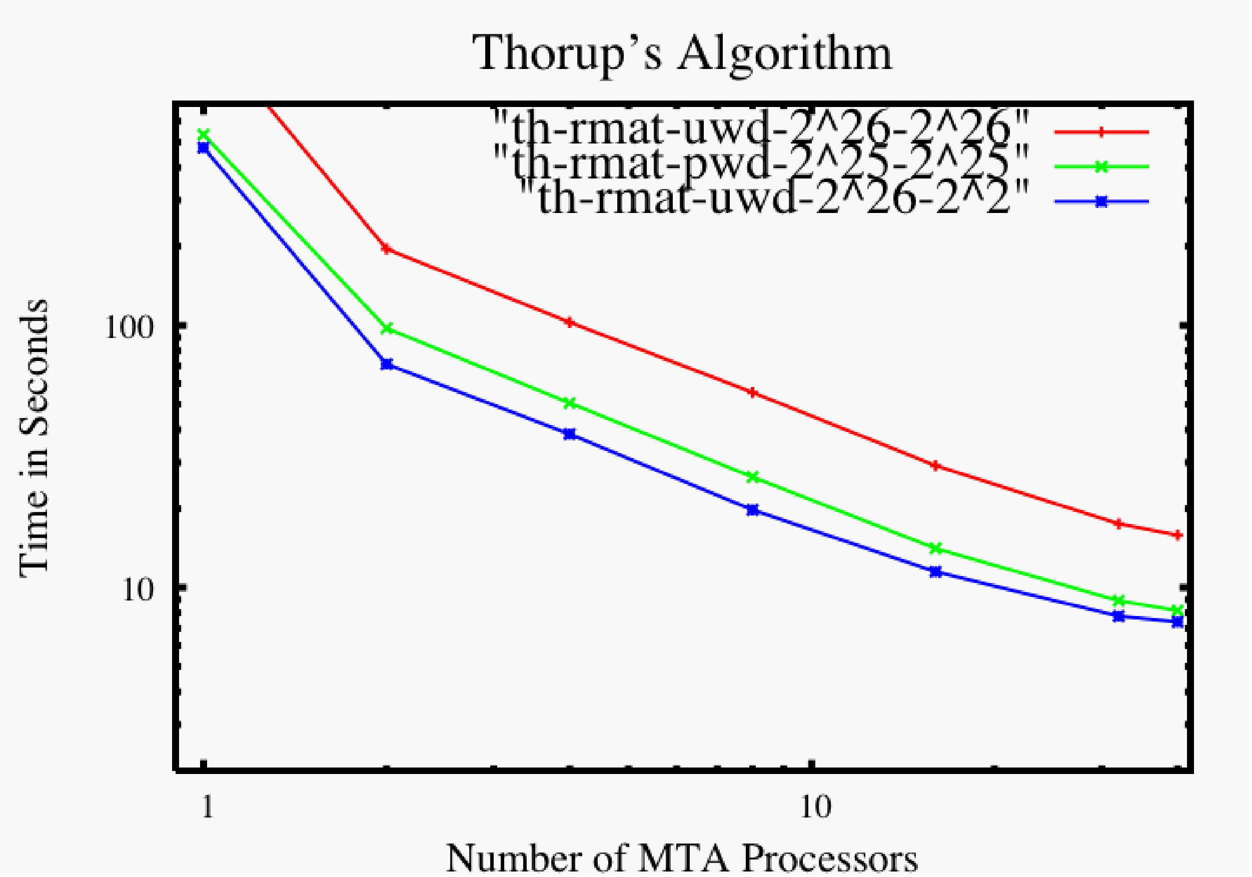
Sequential Performance

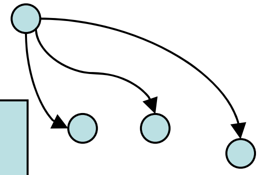
Graph Family	Thorup	DIMACS reference solver
Rand UWD 2^{20} 2^{20}	4.31s	1.66s
Rand UWD 2^{20} 2^{2}	2.66s	1.24s

Random Graph Performance

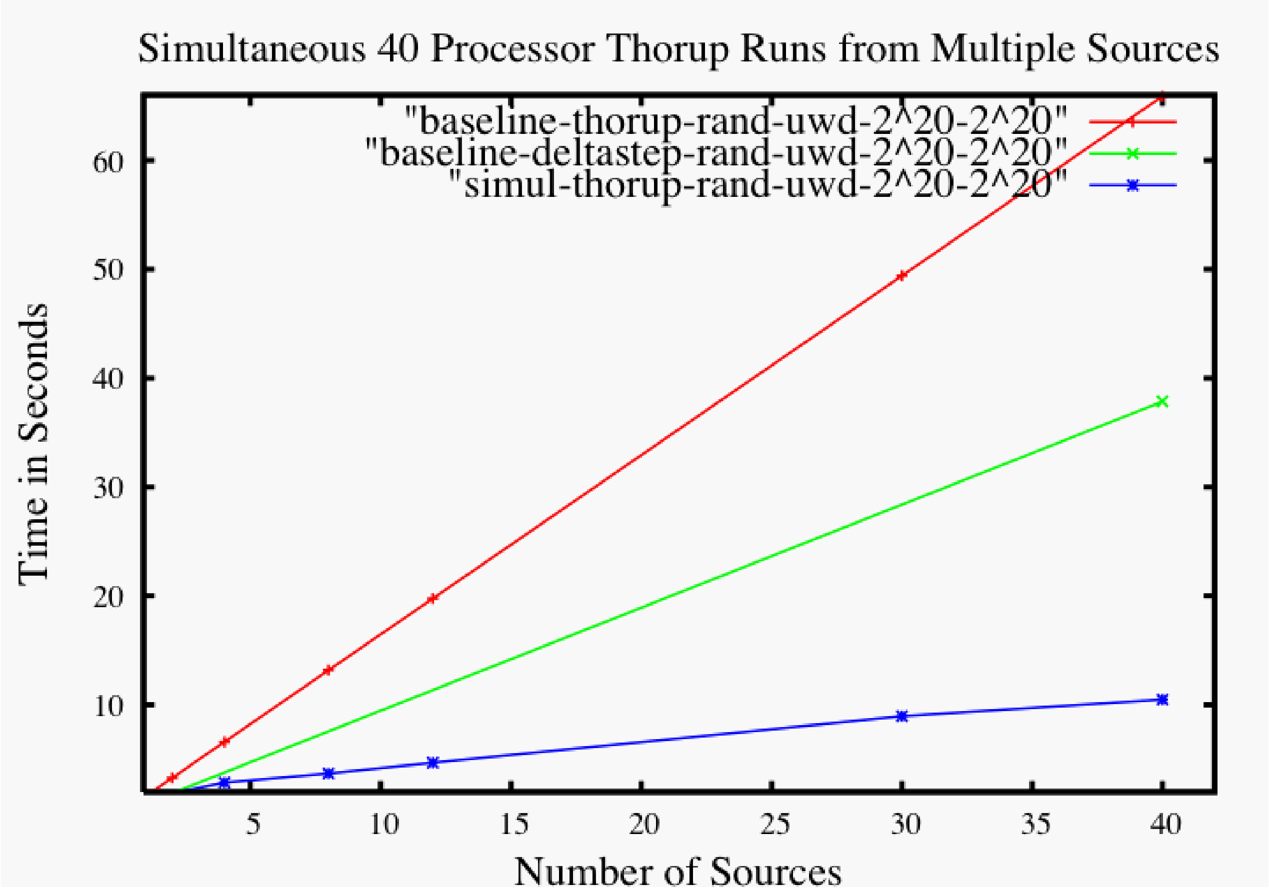


Scale Free Graph Performance





Simultaneous queries with shared CH



Conclusions

- We experimentally evaluate a parallel implementation of Thorup's algorithm for solving SSSP
- Our implementation demonstrates near-linear speedup on several low-diameter graph classes
- Our implementation supports simultaneous SSSP queries
- High diameter graphs are still a challenge to solve in parallel

Future Work

- Perform preprocessing for road networks.
- Phased implementation of Thorup’s algorithm.

Thank You

- Questions?

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