Reducing the Run-time of MCMC Programs by Multithreading on SMP Architectures

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Outline

THE UNIVERSITY OF WARWICK

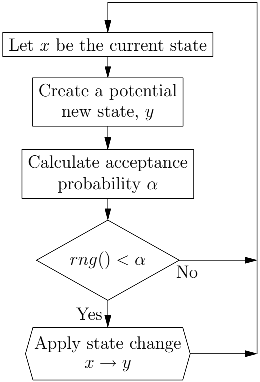
What is Markov Chain Monte Carlo?

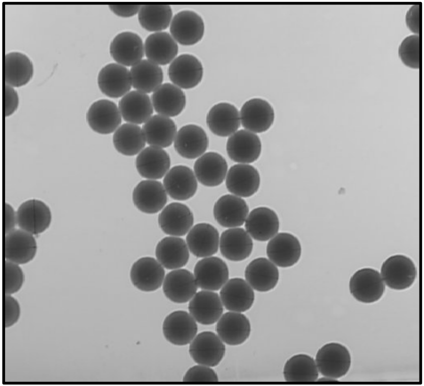
- MCMC is a computationally expensive iterative technique for sampling from a probability distribution.
- Basic idea:
  - Construct a Markov Chain such that its stationary distribution is equal to the distribution we wish to sample.
  - After sufficient burn-in time, sampling from the chain is equivalent to sampling from the distribution.

What uses Markov Chain Monte Carlo?

- MCMC is widely used in
  - Bayesian statistics
  - Computational physics
  - Computational biology
- Specific applications include:
  - Phylogenetic analysis
  - Spectral modelling of X-ray data from the Chandra X-ray satellite
  - Calculating financial econometrics
  - Mapping vascular trees from retinal slides

The MCMC Program Cycle





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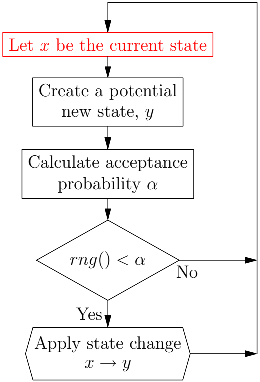

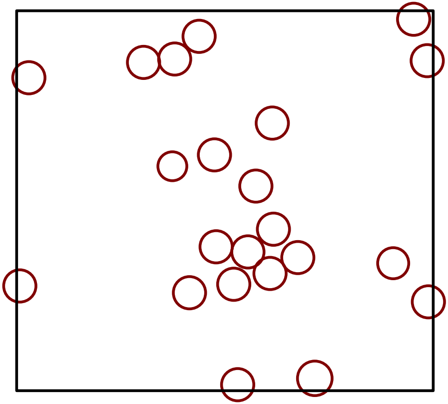

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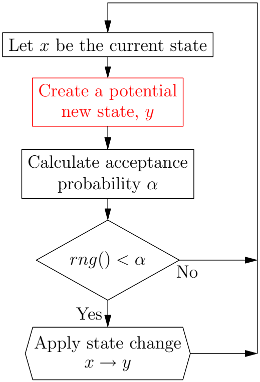

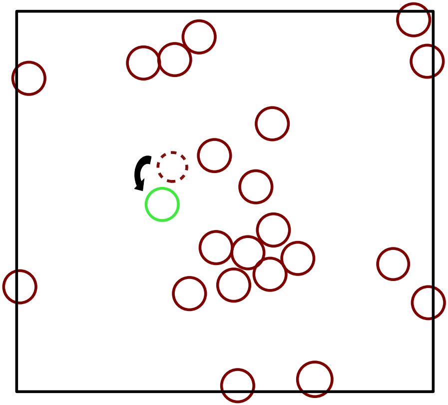

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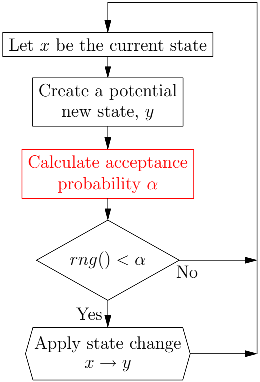

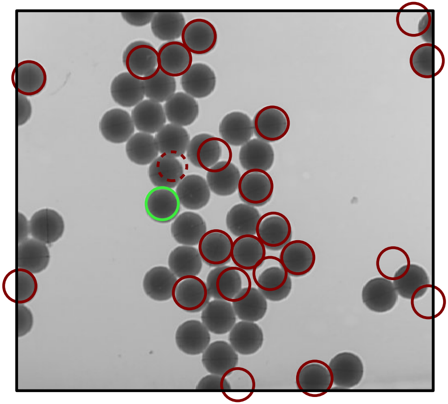

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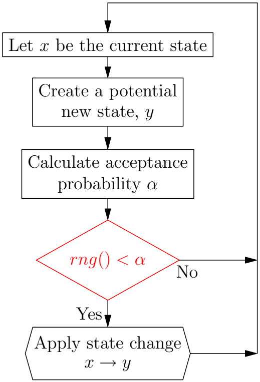

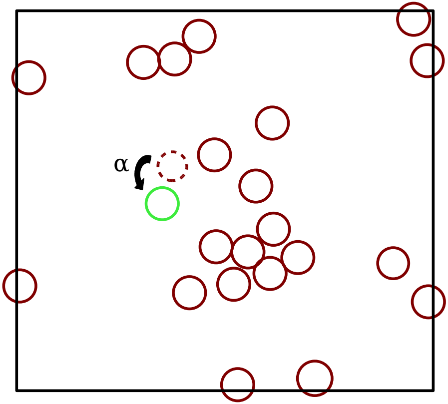

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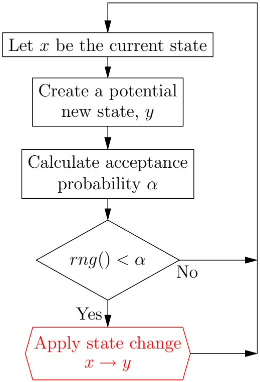

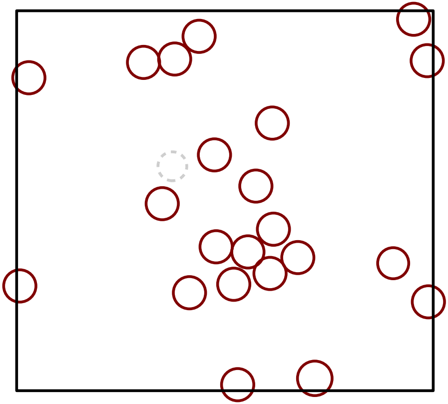

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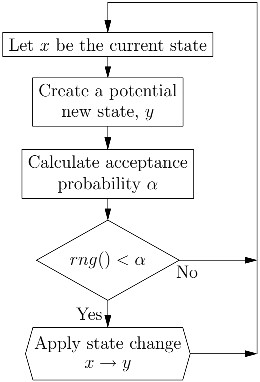

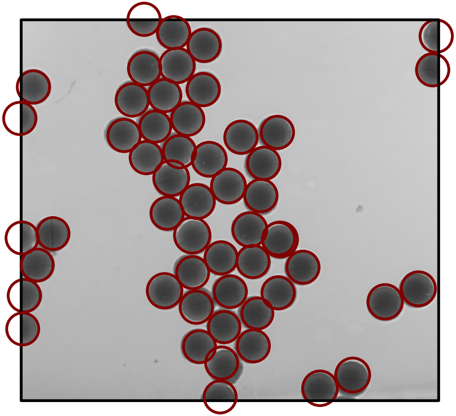

Existing Parallelisation

- Execute multiple chains. Take samples from all of them.
  - Embarrassingly parallel
  - Does not reduce burn-in time.
  - Does not help escape local optima.
- Metropolis-Coupled MCMC
  - Execute multiple chains.
  - Coarse parallelisation, machines connected by LAN.
  - Modifies algorithm to improve rate of convergence.
  - Good for escaping local optima.
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Introducing Speculative Moves

1. Each state in a Markov Chain must depend on only the preceding state.
2. But, typically only $\frac{1}{4}$ of iterations accept the proposed state-change.
3. Consecutive rejected iterations could have been performed in parallel.
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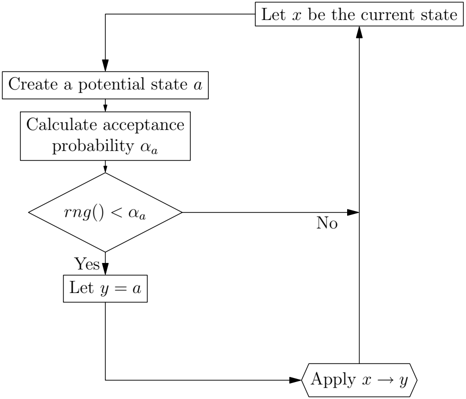

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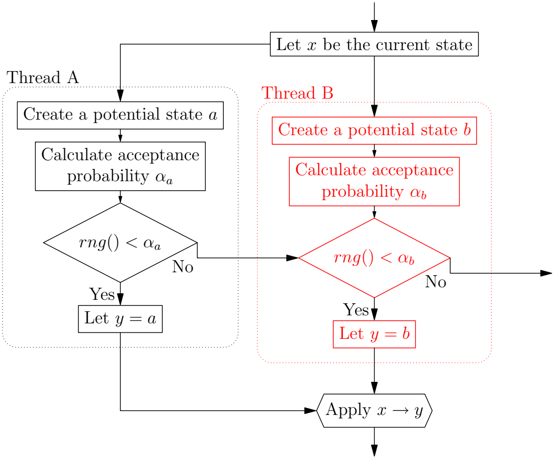

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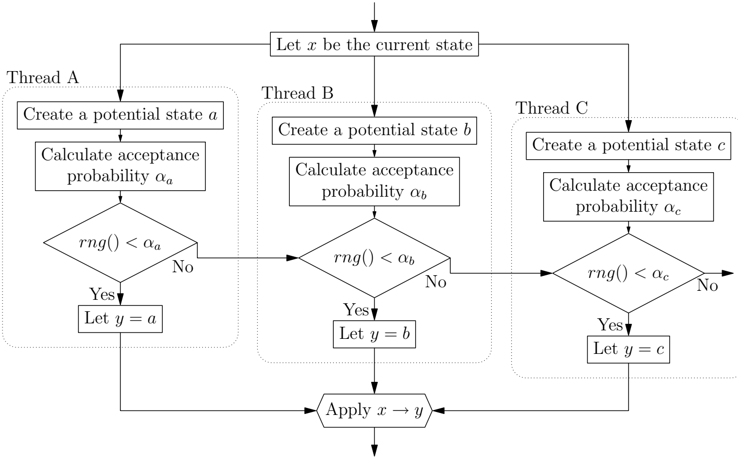
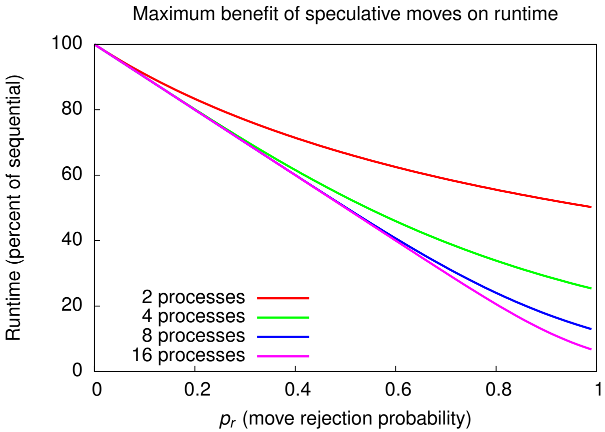


Theoretical Benefits of Speculative Moves

- Let:
  - n be the number of iterations considered concurrently.
  - p_r be the average state-change rejection probability.
- Each program cycle performs $1..n$ MCMC iterations.
- On average $\frac{1-p_r^n}{1-p_r}$ MCMC iterations are performed at each loop of the program cycle.
- If multithreading overhead negligible, time for 1 program cycle $\approx$ time for 1 MCMC iteration.

Theoretical Results



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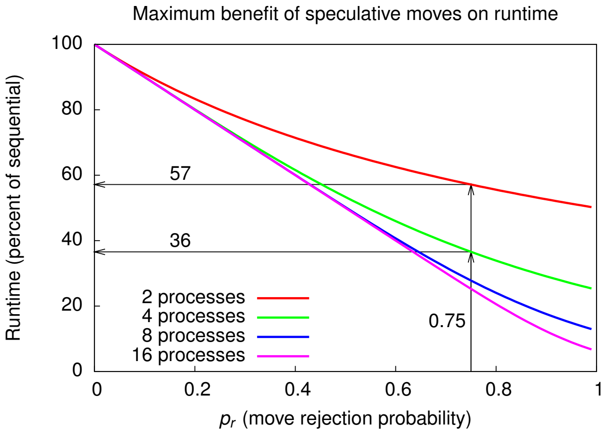
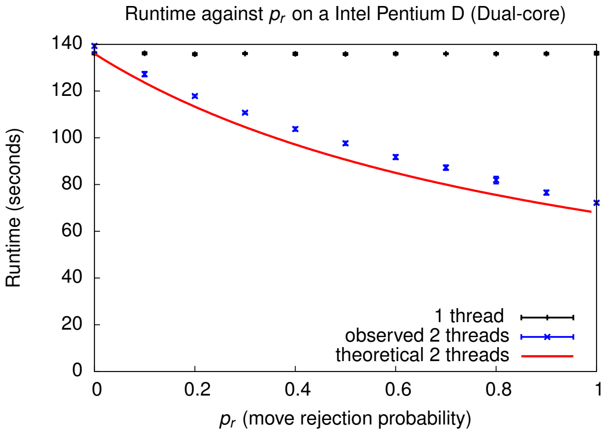


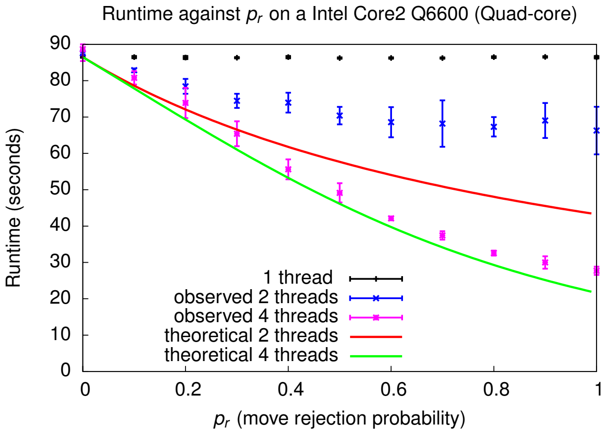
Practical Testing

- Circle detection algorithm used for testing
  - Fixed number of iterations
  - Autogenerated images
  - Runtime values averaged over 20 runs
- Hardware utilised:
  - AMD Athlon 64 X2 4400+ (dual-core)
  - Intel Xeon Dual-Processor
  - Intel Pentium-D (dual core)
  - Intel Core2 Quad Q6600 (2x dual-core dies)
  - 56 Itanium2 processor SGI Altix

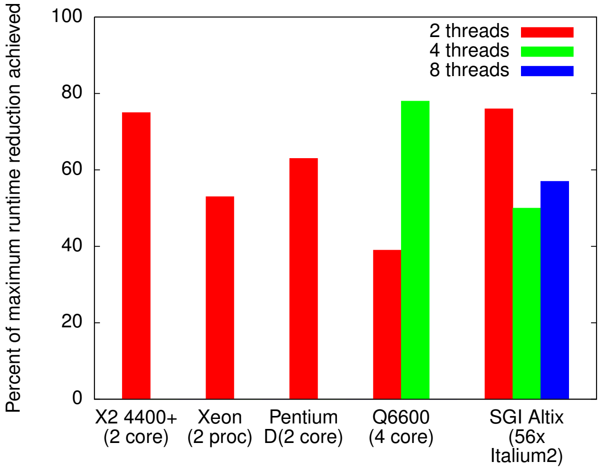
Comparing Practical with Theoretical (1)



Comparing Practical with Theoretical (2)



Preferable Architectures



Will I Benefit?

This table shows the iteration time at which the overhead from multithreading balances the benefits, when $p_r = 0.75$.

	Iteration Time (μs)	Iteration Rate (s^{-1})
Xeon Dual-Processor	70	14 285
Pentium-D (dual core)	55	18 181
Q6600 (using 2 threads)	75	13 333
Q6600 (using 4 threads)	25	40 000

Summary

- The speculative moves method uses increasingly available multiprocessor and multicore machines to reduce the runtime of MCMC program.
- The statistical algorithm is preserved. Speculative moves will not effect the results, only the real-time required to obtain them.
- Real-time reductions of 35% using a dual-core and 55% using quad-cores machines have been demonstrated.