

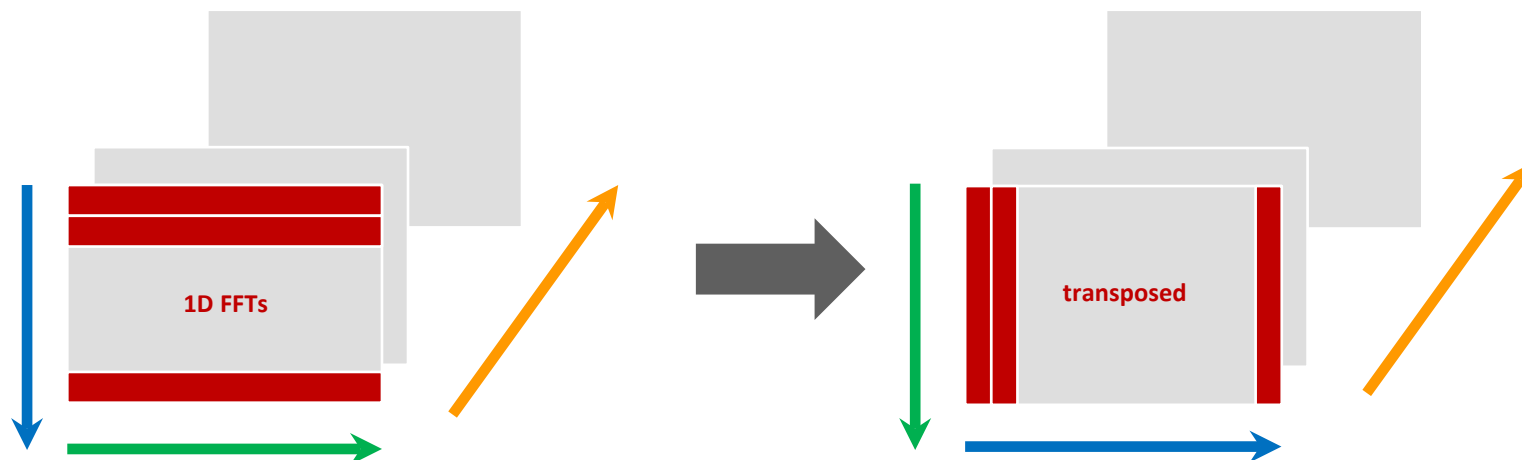
HPC Libraries as DSL

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Supported by the DARPA PERFECT Program

How (Not To) Program a Computational Kernel



```
for (chan = 0; chan < N_CHAN; ++chan)
{
    for (range = 0; range < N_RANGE; ++range)
    {
        /* Apply a 1D FFT to convert to Doppler space */
        fft(
            (float *)&datacube_pulse_major_padded[chan][range][0],
            N_DOP,
            N_DOP_LOG2,
            FFT_FORWARD);
    }
}
/* Transpose from doppler major to range major*/
corner_turn_to_range_major_order
(doppler_datacube, datacube_pulse_major_padded);
```

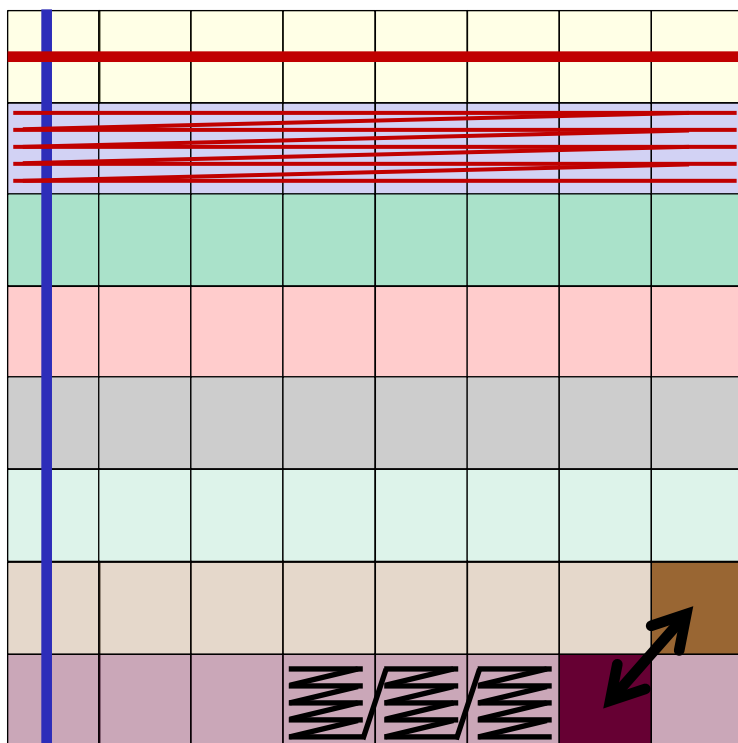
7.5x

```
/* Replace with fftw guru interface*/
plan = fftwf_plan_guru_dft(
    1, dims,
    2, howmany_dims,
    datacube_pulse_major_padded,
    doppler_datacube,
    FFTW_FORWARD,
    FFTW_WISDOM_ONLY);

fftwf_execute(plan);
fftwf_destroy_plan(plan);
```

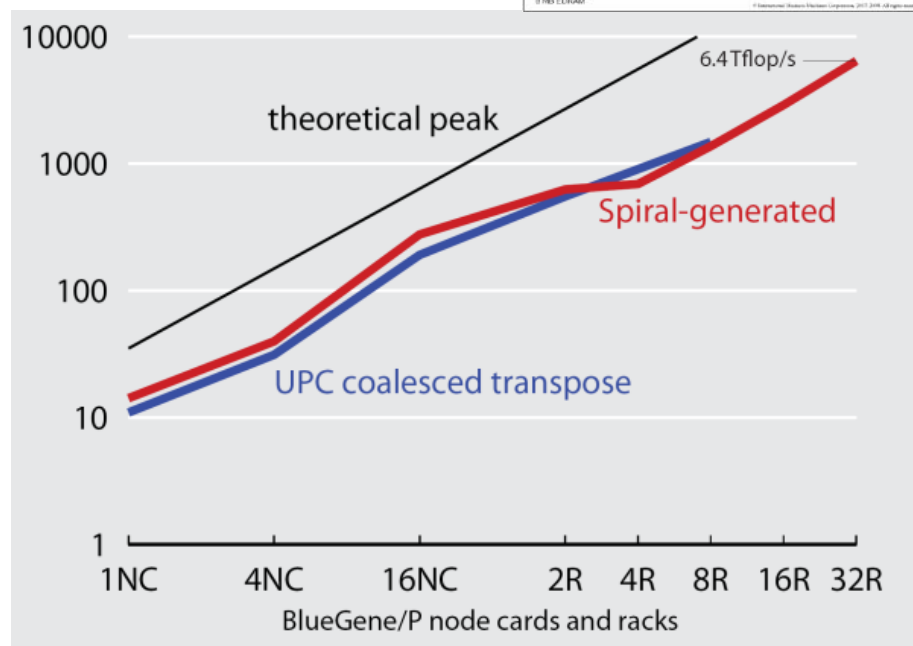
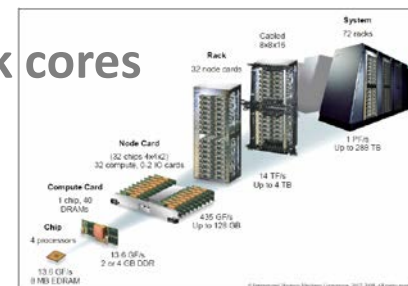
But: Libraries Impose Incompatible Constraints

FFTW: contiguous rows/columns



MPI: contiguous sub-blocks

128k cores



30% gain from MPI-aware FFTW

Using Libraries—Isn't that best HPC practice?

In theory, yes. **But**

- **Sometimes looks like overkill**

Add a whole library for a one-page function?!

- **People often do not use them**

dependencies, don't know them,
“not invented here”

- **Need to combine many libraries**

MPI, FFTW, LAPACK, Boost, STL,...

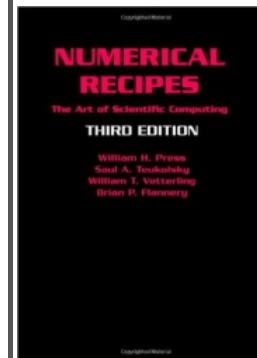
- **Uneven performance and coverage across functions**

not whole FFTW interface well-supported by
MKL, ESSL,...

1 page vs. 30 MB C code

12.2 Fast Fourier Transform (FFT)

507



data[1..2*nn]) is 2 times nn, with each complex value
ive locations. In other words, data[1] is the real part of
inary part of f_0 , and so on up to data[2*nn-1], which
, and data[2*nn], which is the imaginary part of f_{N-1} .
back the F_n 's packed in exactly the same fashion, as nn

ary parts of the zero frequency component F_0 are in data [1]
st nonzero positive frequency F_1 are in data [2*nn+1] and [2*nn+2] in
the smallest (in magnitude) negative frequency F_{-1} is in data [2*nn-1] and data [2*nn]. Positive frequencies
in data [2*nn+1] and data [2*nn]. Positive frequencies
are stored in the real-imaginary pairs data[5], data[6]
data[nn]. Negative frequencies of increasing magnitude are
], data[2*nn-2] down to data[nn+3], data[nn+4].
n+1], data[nn+2] contain the real and imaginary parts of
contains the most positive and the most negative frequency.
pp a familiarity with this storage arrangement of complex
figure 12.2.2, since it is the practical standard.

to remind you that you can also use a routine like four1
if your input data array is zero-offset, that is has the range
data[0..2*nn-1]. In this case, simply decrement the pointer to data by one when
four1 is invoked, e.g., four1(data-1,1024,1). The real part of f_0 will now be
returned in data[0], the imaginary part in data[1], and so on. See §1.2.

```
#include <math.h>
#define SWAP(a,b) tempr=(a);(a)=(b);(b)=tempr

void four1(float data[], unsigned long nn, int isign)
Replaces data[1..2*nn] by its discrete Fourier transform, if isign is input as 1; or replaces
data[1..2*nn] by nn times its inverse discrete Fourier transform, if isign is input as -1.
data is a complex array of length nn or, equivalently, a real array of length 2*nn. nn MUST
be an integer power of 2 (this is not checked for!).
{
    unsigned long n,mmax,m,j,istep,i;
    double wtemp,wr,wpr,wpi,wi,theta;
    float tempr,tempi;

    n=nn << 1;
    j=1;
    for (i=1;i<n;i+=2) {
        if (j > i) {
            SWAP(data[j],data[i]);
            SWAP(data[j+1],data[i+1]);
        }
        m=nn;
        while (m >= 2 && j > m) {
            j -= m;
            m >>= 1;
        }
        j += m;
    }
    Here begins the Danielson-Lanczos section of the routine.
    mmax=2;
    while (n > mmax) {
        istep=mmax << 1;
        theta=isign*(6.28318530717959/mmax);
        wtemp=sin(0.5*theta);
        wpr = -2.0*wtemp*wtemp;
        wpi = 2.0*wtemp;
        Double precision for the trigonomet-
        ric recurrences.

        This is the bit-reversal section of the
        routine.
        Exchange the two complex numbers.

        Outer loop executed log2 nn times.
        Initialize the trigonometric recurrence.
```

Idea 1: Standard HPC Libraries as Kernel DSL

DSL definition and syntax

- **API of standard HPC libraries**

FFTW, LAPACK, BLAS, SparseBLAS, GraphBLAS

- **Libraries and language extensions for parallelism**

OpenMP, MPI, OpenACC

- **Subset of C/C++**

Single threaded, side-effect free, only standard C library calls

DSL semantics and user knowledge

- **C semantics + library semantics**

Program can be executed

- **Use OpenMP/OpenACC annotations to communicate meta-information**

`#pragma omp for private(...) shared(...)`

Idea 2: Interpret Program as Specification

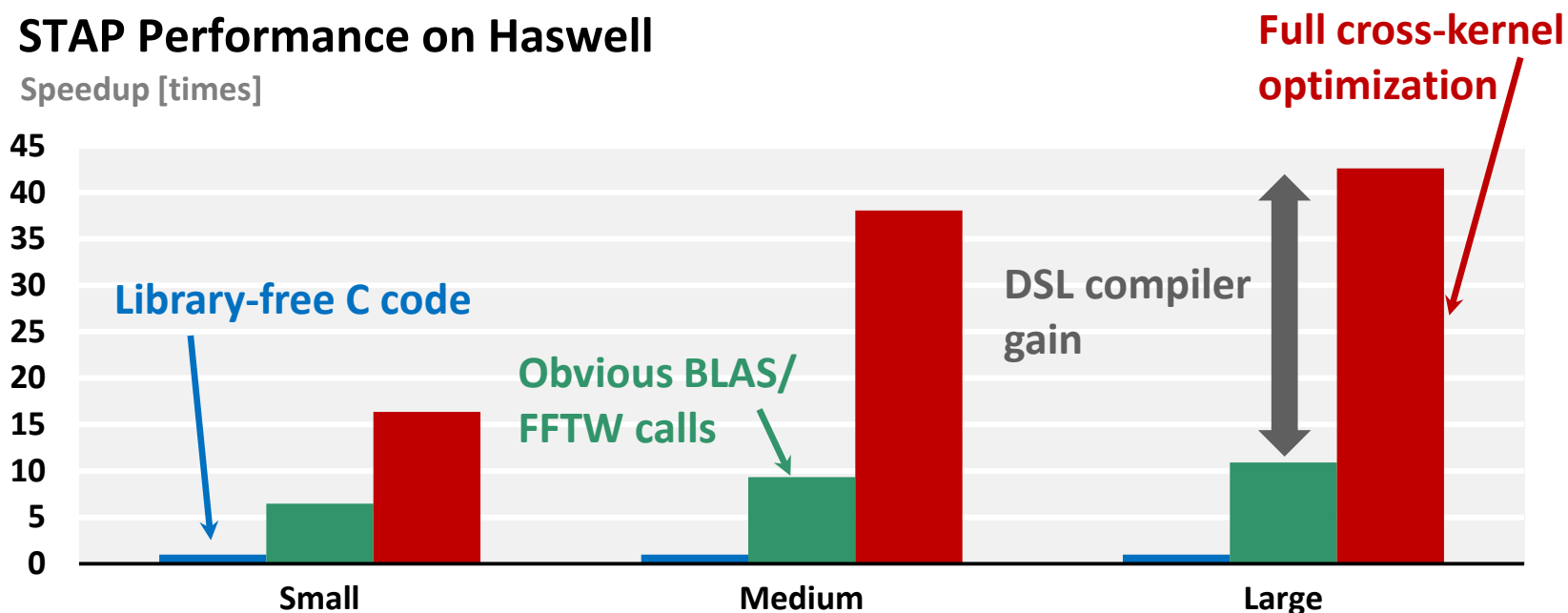
- **Make a *subset* of “C+OpenMP+MKL” a DSL with DSL compiler**
Combines code synthesis, telescoping languages, and HPC best practices
- **Treat DSL program as *specification*, not as program**
computational kernels only, thus moderate code complexity
- **Library calls are a DSL instructions, C fragments are “call-backs”**
overcomes the usual problem of code between library calls
- **Establish side effect-freeness and parallelization opportunities**
use OpenMP/OpenACC to express independent loops, variable visibility,...
- **Enables whole program optimization in domain specific compiler**
data layout transformations, kernel merging, target novel accelerators,...

Result: Performance Portability

- **Same source code, good performance across architectures**
Intel Haswell, Intel Xeon PHI and Near Memory Accelerator
- **Parallel cross-kernel optimized library-based code is fast**
40x speed-up over C baseline (PNNL TAV STAP benchmark)
- **Library-based code is pre-requisite for *Near Memory Accelerator***
130x performance and 8,000x power efficiency gain on accelerator over C base line

STAP Performance on Haswell

Speedup [times]



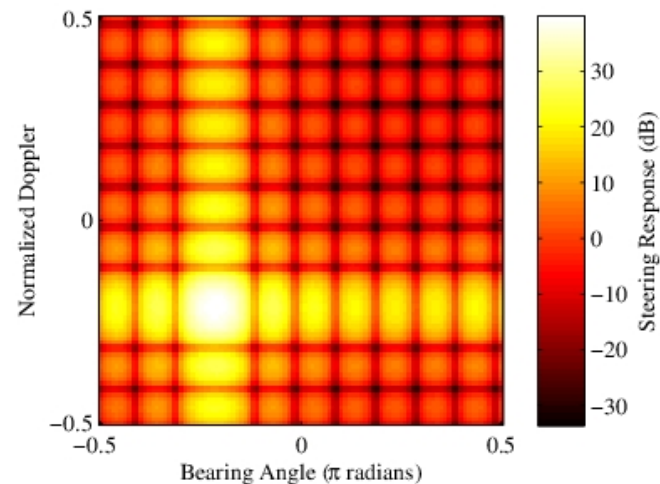
Outline

- **Example: STAP**
- **Cross-call/cross library optimization with Spiral**
- **Library-based hardware acceleration**
- **Summary**

Space Time Adaptive Processing (STAP)



dense data cube



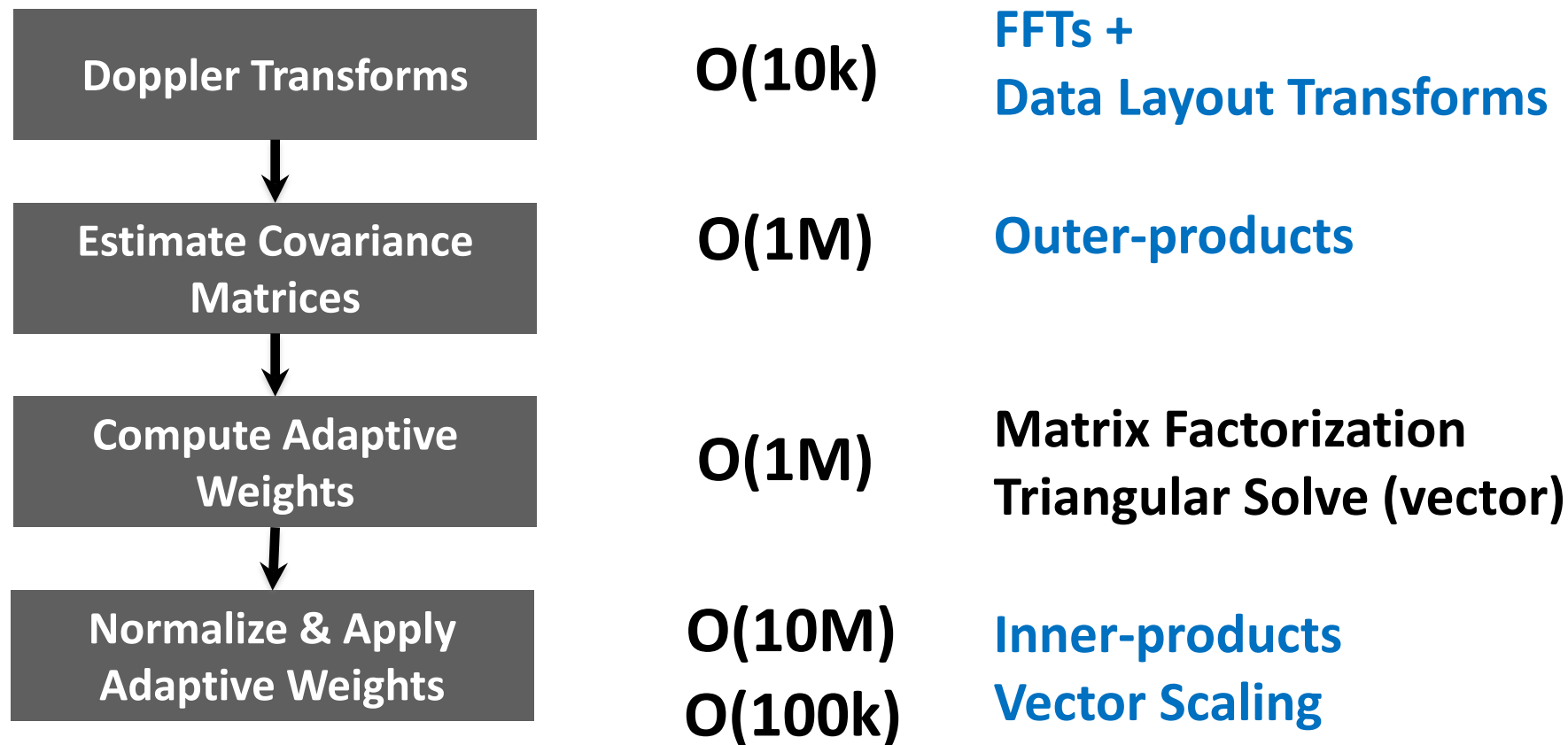
Input: 32 MB

Output: 128 MB

Millions of Math Ops

SWAP requirement: high performance, low power

Main Computational Stages in STAP



Many memory-bounded operations

Background: FFTW 3

- **Latest version of FFTW**
supports threading, SIMD, MPI
- **De-facto standard FFT library for HPC**
distributed with Linux, installed everywhere
- **Autotuning + program generation**
small hand-written core
- **Interface widely supported**
Intel MKL, IBM ESSL, AMD ACML,...

```
#include <fftw3.h>
...
{
    fftw_complex *in, *out;
    fftw_plan p;
    ...
    in = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
    out = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
    p = fftw_plan_dft_1d(N, in, out, FFTW_FORWARD, FFTW_ESTIMATE);
    ...
    fftw_execute(p); /* repeat as needed */
    ...
    fftw_destroy_plan(p);
    fftw_free(in); fftw_free(out);
}
```

FFTW Home Page - Internet Explorer

http://www.fftw.org/

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Introduction

FFTW is a C subroutine library for computing the discrete Fourier transform (DFT) in one or more dimensions, of arbitrary input size, and of both real and complex data (as well as of even/odd data, i.e. the discrete cosine/sine transforms or DCT/DST). We believe that FFTW, which is [free software](#), should become the [FFT](#) library of choice for most applications.

The latest official release of FFTW is version 3.3.4, available from [our download page](#). Version 3.3 introduced support for the AVX x86 extensions, a distributed-memory implementation on top of MPI, and a Fortran 2003 API. Version 3.3.1 introduced support for the ARM Neon extensions. See the [release notes](#) for more information.

The FFTW package was developed at [MIT](#) by [Matteo Frigo](#) and [Steven G. Johnson](#).

Our [benchmarks](#), performed on a variety of platforms, show that FFTW is often faster than other publicly available FFT software, and is even competitive with commercial codes, however, FFTW's performance is *portable*: the same code runs on many different architectures without modification. Hence the name, "FFTW," which stands for "Fastest *T*ransform in the *W*est."

Subscribe to the [fftw-announce mailing list](#) to receive announcements of new releases.

Features

FFTW 3.3.4 is the latest official version of FFTW (see [our download page](#) for some of FFTW's more interesting features):

- [Speed](#). (Supports SSE/SSE2/Altivec, since version 3.0.)
- Both one-dimensional and **multi-dimensional** transforms.
- **Arbitrary-size** transforms. (Sizes with small prime factors are faster.)
- Fast transforms of **purely real** input or output data.
- Transforms of real even/odd data: the [discrete cosine/sine transforms](#) (types I-IV. (Version 3.0 or later.)
- Efficient handling of **multiple, strided** transforms.
- **Parallel transforms**: parallelized code for platforms with OpenMP or MPI.
- **Portable** to any platform with a C compiler.
- [Documentation](#) in HTML and other formats.
- Both C and Fortran interfaces.
- [Free](#) software, released under the GNU General Public License (GPL). (See also the [FAQ](#).)

If you are still using FFTW 2.x, please note that FFTW 3.x is **incompatible** with 2.x (see the [FAQ](#) or the [manual](#)).

PROCEEDINGS OF THE IEEE

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Special Issue on:

PROGRAM GENERATION, OPTIMIZATION, AND PLATFORM ADAPTATION

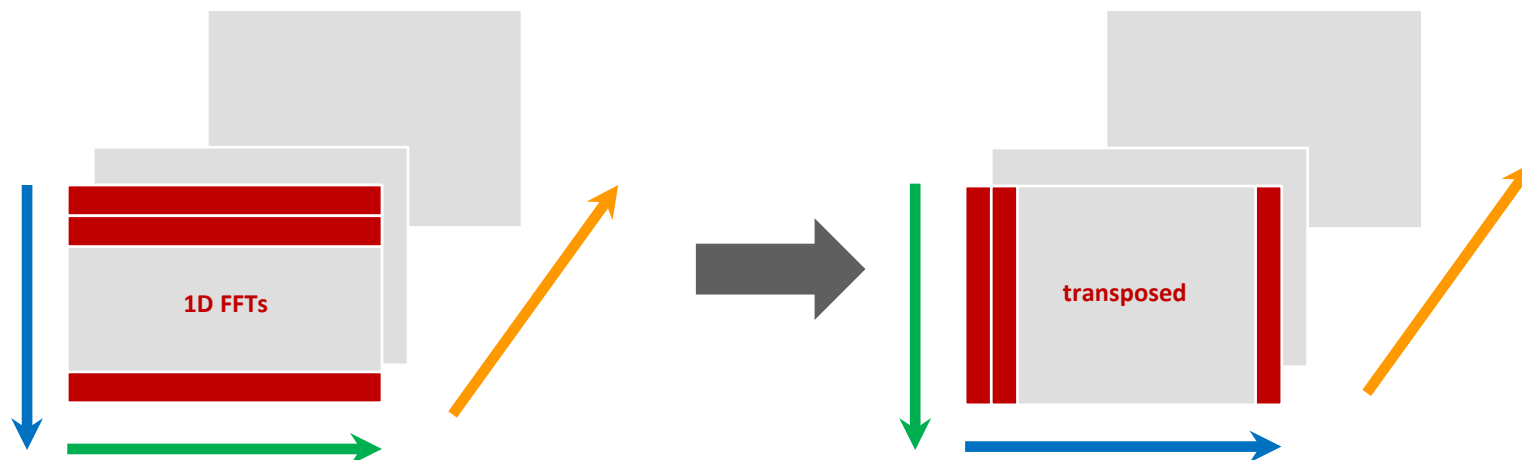
Papers on:

- Design & Implementation of FFTW3 • SPiRAL: Code Generation for DSP Transforms
- Synthesis of Parallel Programs for Ab Initio Quantum Chemistry Models • Self-Adapting Linear Algebra Algorithms & Software • Parallel VSPL++: An Open Standard for Parallel Signal Processing • Parallel MAXLAI: Doing it Right • Broadway: Exploiting the Domain-Specific Semantics of Software Libraries • Is Search Really Necessary to Generate High-Performance BLAS? • Telescoping Languages: Automatic Generation of Domain Languages
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Scanning Our Past: Electrical Engineering Hall of Fame: Alexander Graham Bell

IEEE

Doppler Transform: FFT + Corner Turn



```
for (chan = 0; chan < N_CHAN; ++chan)
{
    for (range = 0; range < N_RANGE; ++range)
    {
        /* Apply a 1D FFT to convert to Doppler space */
        fft(
            (float *)&datacube_pulse_major_padded[chan][range][0],
            N_DOP,
            N_DOP_LOG2,
            FFT_FORWARD);
    }
}
/* Transpose from doppler major to range major*/
corner_turn_to_range_major_order
(doppler_datacube, datacube_pulse_major_padded);
```

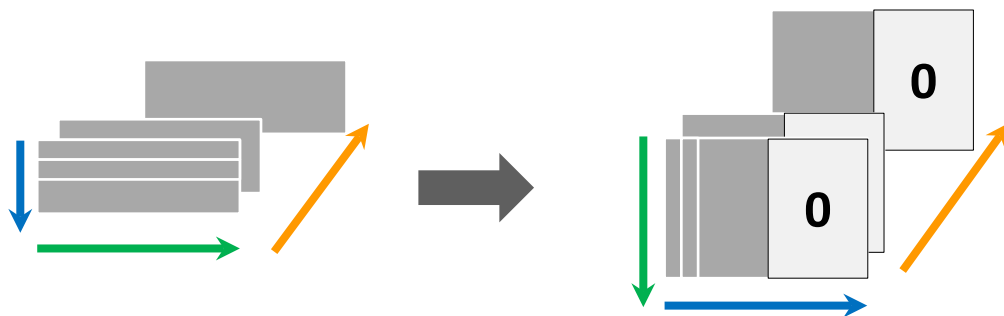
```
/* Replace with fftw guru interface*/
plan = fftwf_plan_guru_dft(
    1, dims,
    2, howmany_dims,
    datacube_pulse_major_padded,
    doppler_datacube,
    FFTW_FORWARD,
    FFTW_WISDOM_ONLY);

fftwf_execute(plan);
fftwf_destroy_plan(plan);
```

Combine loop of FFTs plus subsequent corner turn into one FFTW call

Use FFTW Guru Interface for Data Copy

Zero pad plus batch transpose

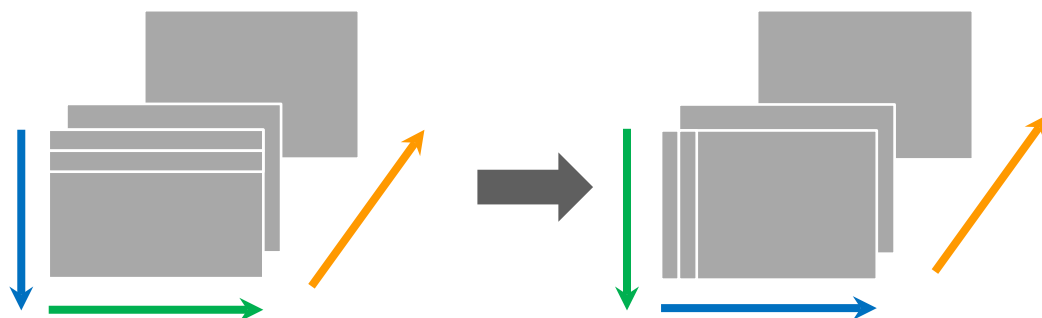


```
const fftwf_iodim howmany_dims[3] =
{{n:N_RANGE, is:1, os:N_DOP},
 {n:N_PULSES, is:N_RANGE, os:1},
 {n:N_CHAN,
  is:N_RANGE*N_PULSES,
  os:N_RANGE*N_DOP}}};

plan =
fftwf_plan_guru_dft(0, NULL,
                    3, howmany_dims,
                    datacube,
                    datacube_pulse_major_padded,
                    FFTW_FORWARD,
                    FFTW_ESTIMATE);

fftwf_execute(plan);
```

Batch transpose



```
const fftwf_iodim howmany_dims[3] =
{{n:N_DOP, is:1, os:N_RANGE},
 {n:N_RANGE, is:N_DOP, os:1},
 {n:N_CHAN,
  is:N_RANGE*N_DOP,
  os:N_RANGE*N_DOP}}};

plan =
fftwf_plan_guru_dft(0, NULL,
                    3, howmany_dims,
                    doppler_major,
                    range_major,
                    FFTW_FORWARD,
                    FFTW_ESTIMATE);

fftwf_execute(plan);
```

FFTW Rank-0 FFT abstracts copy, transpose, gather, scatter

Borrowing FFTW's Guru Interface for BLAS

■ FFTW guru interface

strong support for batch FFTs and n-dimensional data cubes

```
typedef struct{
    int n; // size of the dimension
    int is; // stride for input
    int os; // stride for output
} fftw_iodim;
```

```
/* FFTW Guru Interface */
fftw_plan fftw_plan_guru_dft(
    int rank, const fftw_iodim *dims,
    int howmany_rank, const fftw_iodim *howmany_dims,
    fftw_complex *in, fftw_complex *out,
    int sign, unsigned flags);
```

■ Standard BLAS interface

Fortran 77 style interface

```
void cblas_cdotc_sub (const int N, const void * x, const int incx,
    const void * y, const int incy, void * dotc);
```

■ Generalization: BLAS guru interface

borrow FFTW's data and batch representation for BLAS operations

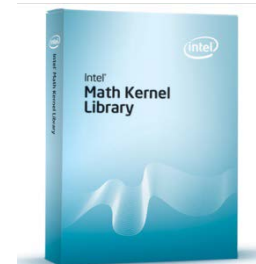
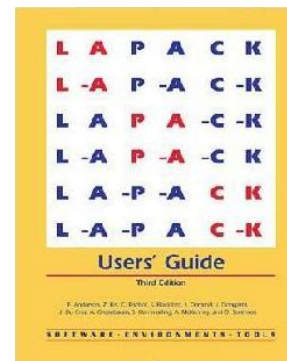
```
typedef struct{
    int n;
    int i0s; // stride for input0
    int i1s; // stride for input1
} blas_indim;
```

```
/* BLAS Guru Configuration Interface */
blas_conf blas_conf_guru(
    // (rank, dims[rank]) describes the basic BLAS operation size
    int rank,
    blas_indim *dims,
    // (howmany_rank, howmany_dims[rank]) describes the "vector" size
    int howmany_rank,
    blas_indim *howmany_dims);
```

Use BLAS guru interface in backend but do not expose to end user

Use OpenMP to Avoid Changing BLAS Interface

- **Absolutely cannot change BLAS**
set-in-stone standard since 1979
- **Supported by everybody**
MKL, ESSL, ACML, CUBLAS,...
- **Idea: Use OpenMP to express batch**
mark loops as independent
- **Use compiler to extract batch BLAS descriptor**
automatically translate BLAS + OpenMP to BLAS Guru Interface



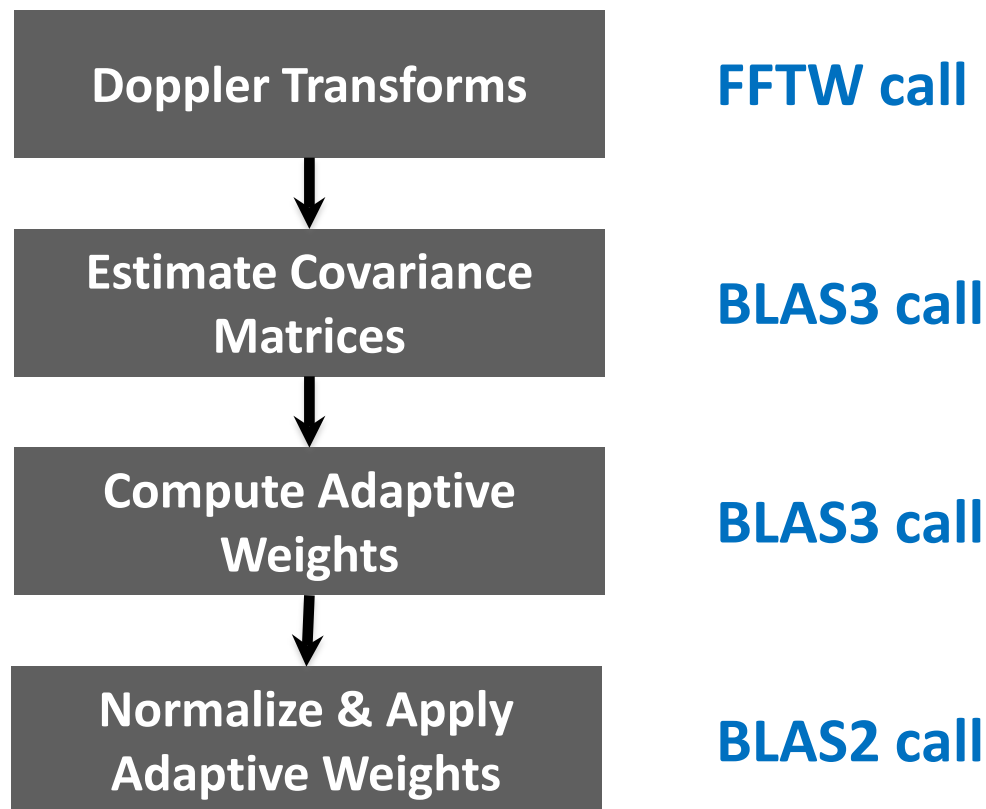
```
#pragma omp for
for (sv = 0; sv < N_STEERING; ++sv) {
#pragma omp for
  for (block = 0; block < N_DOP*N_BLOCKS; ++block) {
    cblas_cdotc_sub(TDOF*N_CHAN,
      (float*)adaptive_weights[block][sv],
      1,
      (float*)steering_vectors[sv],
      1,
      (float*)&accums[block][sv]);
  }
}
```

STAP rewritten with BLAS + OpenMP

compiler

```
blas_guru(
  1,
  {{n:TDOF*N_CHAN, i0s:1, i1s:1}}
  2,
  {{n:N_DOP*N_BLOCKS,
    i0s:N_STEERING*TDOF*N_CHAN,
    i1s:1}},
  {n:N_STEERING,
    i0s:TDOF*N_CHAN,
    i1s:TDOF*N_CHAN}});
```

After Transformation: STAP = 4 Library Calls



Result: STAP is expressed using four library calls + OpenMP directives

Outline

- Example: STAP
- **Cross-call/cross library optimization with Spiral**
- Library-based hardware acceleration
- Summary

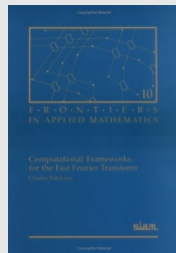
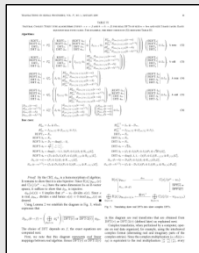
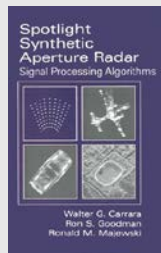
DSL Compiler for Global Transformations

- Parse library calls and OpenMP convert to Spiral's operator language (OL)
- **Spiral performs global optimizations** kernel merging, data layout transformations,...
- **Output CPU or accelerator code** run with Intel MKL or our own accelerator
- **Utilize BLAS Guru interface** our accelerator implements BLAS Guru



What is Spiral?

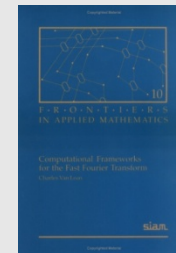
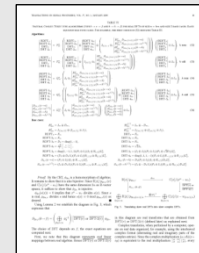
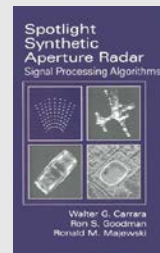
Traditionally



High performance library
optimized for given platform

*Comparable
performance*

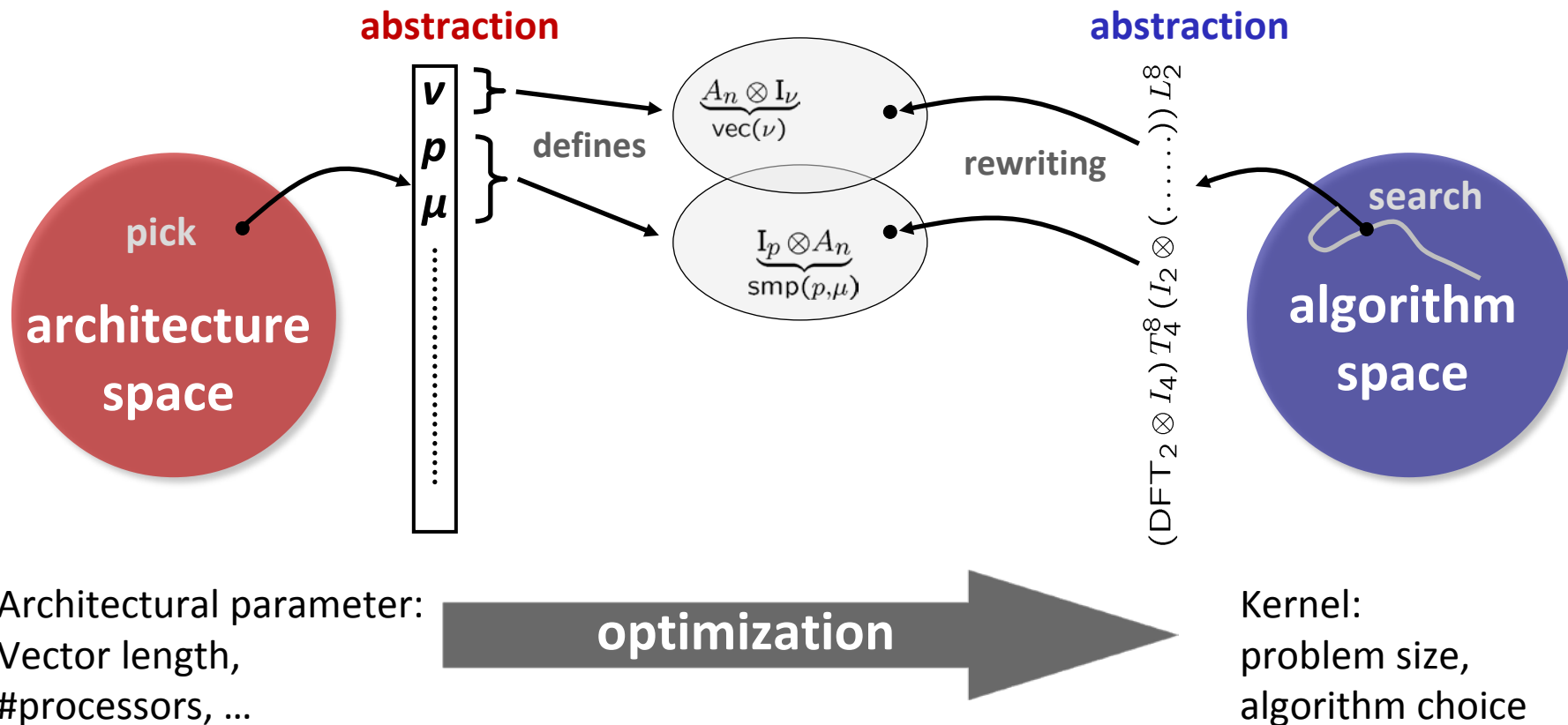
Spiral Approach



High performance library
optimized for given platform

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas



Operators

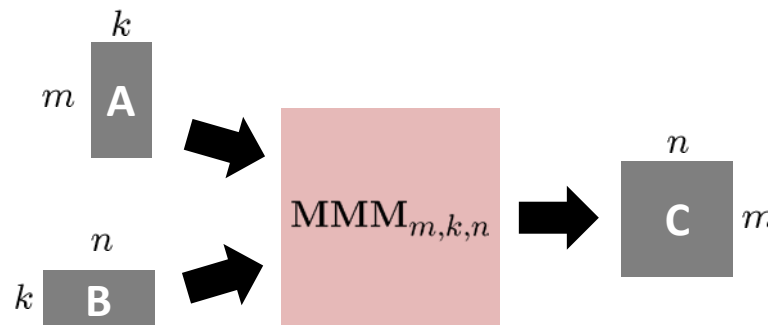
Definition

- Operator: Multiple complex vectors \rightarrow multiple complex vectors
- Higher-dimensional data is linearized
- Operators are potentially nonlinear

$$M : \begin{cases} \mathbb{C}^{n_0} \times \dots \times \mathbb{C}^{n_{k-1}} \rightarrow \mathbb{C}^{N_0} \times \dots \times \mathbb{C}^{N_{\ell-1}} \\ (\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{k-1}) \mapsto M(\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{k-1}) \end{cases}$$

Example: Matrix-matrix-multiplication (MMM)

$$\text{MMM}_{m,k,n} : \mathbb{R}^{mk} \times \mathbb{R}^{kn} \rightarrow \mathbb{R}^{mn}$$
$$(A, B) \mapsto AB$$



Key to capture FFTs, numerical linear algebra, message passing in one framework

Operator Language

name	definition
<i>Linear, arity (1,1)</i>	
identity	$I_n : \mathbb{C}^n \rightarrow \mathbb{C}^n; \mathbf{x} \mapsto \mathbf{x}$
vector flip	$J_n : \mathbb{C}^n \rightarrow \mathbb{C}^n; (x_i) \mapsto (x_{n-i})$
transposition of an $m \times n$ matrix	$L_m^{mn} : \mathbb{C}^{mn} \rightarrow \mathbb{C}^{mn}; \mathbf{A} \mapsto \mathbf{A}^T$
matrix $M \in \mathbb{C}^{m \times n}$	$M : \mathbb{C}^n \rightarrow \mathbb{C}^m; \mathbf{x} \mapsto M\mathbf{x}$
<i>Multilinear, arity (2,1)</i>	
Point-wise product	$P_n : \mathbb{C}^n \times \mathbb{C}^n \rightarrow \mathbb{C}^n; ((x_i), (y_i)) \mapsto (x_i y_i)$
Scalar product	$S_n : \mathbb{C}^n \times \mathbb{C}^n \rightarrow \mathbb{C}; ((x_i), (y_i)) \mapsto \sum (x_i y_i)$
Kronecker product	$K_{m \times n} : \mathbb{C}^m \times \mathbb{C}^n \rightarrow \mathbb{C}^{mn}; ((x_i), \mathbf{y})) \mapsto (x_i \mathbf{y})$
<i>Others</i>	
Fork	$\text{Fork}_n : \mathbb{C}^n \rightarrow \mathbb{C}^n \times \mathbb{C}^n; \mathbf{x} \mapsto (\mathbf{x}, \mathbf{x})$
Split	$\text{Split}_n : \mathbb{C}^n \rightarrow \mathbb{C}^{n/2} \times \mathbb{C}^{n/2}; \mathbf{x} \mapsto (\mathbf{x}^U, \mathbf{x}^L)$
Concatenate	$\oplus_n : \mathbb{C}^n \times \mathbb{C}^m \rightarrow \mathbb{C}^{n+m}; (\mathbf{x}, \mathbf{y}) \mapsto \mathbf{x} \oplus \mathbf{y}$
Duplication	$\text{dup}_n^m : \mathbb{C}^n \rightarrow \mathbb{C}^{nm}; (\mathbf{x} \mapsto \mathbf{x} \otimes I_m$
Min	$\text{min}_n : \mathbb{C}^n \times \mathbb{C}^n \rightarrow \mathbb{C}^n; (\mathbf{x}, \mathbf{y}) \mapsto (\min(x_i, y_i))$
Max	$\text{max}_n : \mathbb{C}^n \times \mathbb{C}^n \rightarrow \mathbb{C}^n; (\mathbf{x}, \mathbf{y}) \mapsto (\max(x_i, y_i))$

Some Application Domains in OL

Linear Transforms

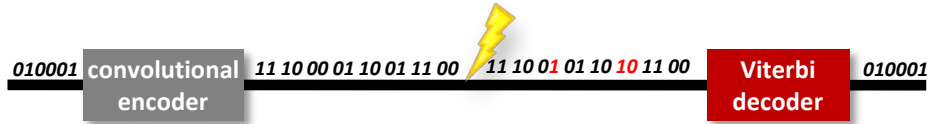
$$\begin{aligned}
 \text{DFT}_n &\rightarrow (\text{DFT}_k \otimes \text{I}_m) \text{T}_m^n (\text{I}_k \otimes \text{DFT}_m) \text{L}_k^n, \quad n = km \\
 \text{DFT}_n &\rightarrow P_n (\text{DFT}_k \otimes \text{DFT}_m) Q_n, \quad n = km, \quad \gcd(k, m) = 1 \\
 \text{DFT}_p &\rightarrow R_p^T (\text{I}_1 \oplus \text{DFT}_{p-1}) D_p (\text{I}_1 \oplus \text{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
 \text{DCT-3}_n &\rightarrow (\text{I}_m \oplus \text{J}_m) \text{L}_m^n (\text{DCT-3}_m(1/4) \oplus \text{DCT-3}_m(3/4)) \\
 &\quad \cdot (\text{F}_2 \otimes \text{I}_m) \begin{bmatrix} \text{I}_m & 0 \oplus -\text{J}_{m-1} \\ \frac{1}{\sqrt{2}}(\text{I}_1 \oplus 2\text{I}_m) \end{bmatrix}, \quad n = 2m \\
 \text{DCT-4}_n &\rightarrow S_n \text{DCT-2}_n \text{diag}_{0 \leq k < n} (1/(2 \cos((2k+1)\pi/4n))) \\
 \text{IMDCT}_{2m} &\rightarrow (\text{J}_m \oplus \text{I}_m \oplus \text{I}_m \oplus \text{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \text{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \text{I}_m \right) \right) \text{J}_{2m} \text{DCT-4}_{2m} \\
 \text{WHT}_{2^k} &\rightarrow \prod_{i=1}^t (\text{I}_{2^{k_1+\dots+k_{i-1}}} \otimes \text{WHT}_{2^{k_i}} \otimes \text{I}_{2^{k_{i+1}+\dots+k_t}}), \quad k = k_1 + \dots + k_t \\
 \text{DFT}_2 &\rightarrow \text{F}_2 \\
 \text{DCT-2}_2 &\rightarrow \text{diag}(1, 1/\sqrt{2}) \text{F}_2 \\
 \text{DCT-4}_2 &\rightarrow \text{J}_2 \text{R}_{13\pi/8}
 \end{aligned}$$

Matrix-Matrix Multiplication

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \times \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$\begin{aligned}
 \text{MMM}_{1,1,1} &\rightarrow (\cdot)_1 \\
 \text{MMM}_{m,n,k} &\rightarrow (\otimes)_{m/m_b \times 1} \otimes \text{MMM}_{m_b,n,k} \\
 \text{MMM}_{m,n,k} &\rightarrow \text{MMM}_{m,n_b,k} \otimes (\otimes)_{1 \times n/n_b} \\
 \text{MMM}_{m,n,k} &\rightarrow ((\sum_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \text{MMM}_{m,n,k_b}) \circ \\
 &\quad ((L_{k/k_b}^{mk/k_b} \otimes \text{I}_{k_b}) \times \text{I}_{kn}) \\
 \text{MMM}_{m,n,k} &\rightarrow (L_m^{mn/n_b} \otimes \text{I}_{n_b}) \circ \\
 &\quad ((\otimes)_{1 \times n/n_b} \otimes \text{MMM}_{m,n_b,k}) \circ \\
 &\quad (\text{I}_{km} \times (L_{n/n_b}^{kn/n_b} \otimes \text{I}_{n_b}))
 \end{aligned}$$

Software Defined Radio

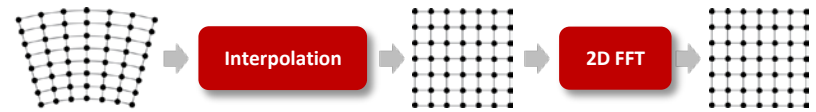


$$\mathbf{F}_{K,F} \rightarrow \prod_{i=1}^F \left((\text{I}_{2^{K-2}} \otimes_j B_{F-i,j}) L_{2^{K-2}}^{2^{K-1}} \right)$$

$$\mathbf{F}_{K,F} \nu \rightarrow \prod_{i=1}^F \left((\text{I}_{2^{K-2}/\nu} \otimes_{j_1} \tilde{\text{L}}_\nu^{2\nu} \tilde{B}_{F-i,j_1}^\nu) (\text{L}_{2^{K-2}/\nu}^{2^{K-1}/\nu} \bar{\otimes} \text{I}_\nu) \right)$$

$$B_{i,j} : \begin{cases} \pi_U = \min_{d_U} (\pi_A + \beta_{A \rightarrow U}, \pi_B + \beta_{B \rightarrow U}) \\ \pi_V = \min_{d_V} (\pi_A + \beta_{A \rightarrow V}, \pi_B + \beta_{B \rightarrow V}) \end{cases}$$

Synthetic Aperture Radar (SAR)



$$\text{SAR}_{k \times m \rightarrow n \times n} \rightarrow \text{DFT}_{n \times n} \circ \text{Interp}_{k \times m \rightarrow n \times n}$$

$$\text{DFT}_{n \times n} \rightarrow (\text{DFT}_n \otimes \text{I}_n) \circ (\text{I}_n \otimes \text{DFT}_n)$$

$$\text{Interp}_{k \times m \rightarrow n \times n} \rightarrow (\text{Interp}_{k \rightarrow n} \otimes_i \text{I}_n) \circ (\text{I}_k \otimes_i \text{Interp}_{m \rightarrow n})$$

$$\text{Interp}_{r \rightarrow s} \rightarrow \left(\bigoplus_{i=0}^{n-2} \text{InterpSeg}_k \right) \oplus \text{InterpSegPruned}_{k,\ell}$$

$$\text{InterpSeg}_k \rightarrow G_f^{u \cdot n \rightarrow k} \circ \text{iPrunedDFT}_{n \rightarrow u \cdot n} \circ \left(\frac{1}{n} \right) \circ \text{DFT}_n$$

Formal Approach for all Types of Parallelism

- **Multithreading** (Multicore)
- **Vector SIMD** (SSE, VMX/Altivec,...)
- **Message Passing** (Clusters, MPP)
- **Streaming/multibuffering** (Cell)
- **Graphics Processors** (GPUs)
- **Gate-level parallelism** (FPGA)
- **HW/SW partitioning** (CPU + FPGA)

$$I_p \otimes_{\parallel} A_{\mu n}, \quad L_m^{mn} \bar{\otimes} I_{\mu}$$

$$A \hat{\otimes} I_{\nu} \quad \underbrace{L_2^{2\nu}}_{\text{isa}}, \quad \underbrace{L_{\nu}^{2\nu}}_{\text{isa}}, \quad \underbrace{L_{\nu}^{\nu^2}}_{\text{isa}}$$

$$I_p \otimes_{\parallel} A_n, \quad \underbrace{L_p^{p^2} \bar{\otimes} I_{n/p^2}}_{\text{all-to-all}}$$

$$I_n \otimes_2 A_{\mu n}, \quad L_m^{mn} \bar{\otimes} I_{\mu}$$

$$\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} I_w, \quad P_n \otimes Q_w$$

$$\prod_{i=0}^{n-1} A_i^{\text{ir}}, \quad I_s \tilde{\otimes} A, \quad \underbrace{L_n^m}_{\text{bram}}$$

$$\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$$

Autotuning in Constraint Solution Space

Intel Core i7 (2nd Gen)



Base cases

$I_4 \otimes_{||} A_{16n}$
 $L_m^{mn} \otimes I_{16}$
 $A \otimes I_4$
 $L_2^8, L_4^8, L_4^{16}, L_2^4 \otimes I_2$
 SSE SSE SSE SSE

Transformation rules

$$\begin{aligned}
 \frac{AB}{\text{smp}(p,\mu)} &\rightarrow \frac{A}{\text{smp}(p,\mu)} \frac{B}{\text{smp}(p,\mu)} \\
 \frac{L_m^{mn}}{\text{smp}(p,\mu)} &\rightarrow \left\{ \begin{array}{l} \left(\frac{I_p \otimes L_{m/p}^{mn/p}}{\text{smp}(p,\mu)} \right) \left(\frac{L_p^{pn} \otimes I_{m/p}}{\text{smp}(p,\mu)} \right) \\ \left(\frac{L_m^{pm} \otimes I_{n/p}}{\text{smp}(p,\mu)} \right) \left(\frac{I_p \otimes L_m^{mn/p}}{\text{smp}(p,\mu)} \right) \end{array} \right. \\
 \frac{I_m \otimes A_n}{\text{smp}(p,\mu)} &\rightarrow I_p \otimes_{||} \left(\frac{I_{m/p} \otimes A_n}{\text{smp}(p,\mu)} \right) \\
 &\dots
 \end{aligned}$$

DFT_{256}



Breakdown rules

$$\begin{aligned}
 \text{DFT}_n &\rightarrow (\text{DFT}_k \otimes I_m) T_m^n \\
 &\quad \cdot (I_k \otimes \text{DFT}_m) L_k^n \\
 \text{DFT}_n &\rightarrow P_n (\text{DFT}_k \otimes \text{DFT}_m) Q_n \\
 \text{DFT}_p &\rightarrow R_p^T (I_1 \oplus \text{DFT}_{p-1}) D_p \\
 &\quad \cdot (I_1 \oplus \text{DFT}_{p-1}) R_p \\
 \text{DFT}_2 &\rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
 \end{aligned}$$

Expansion + backtracking

OL specification

OL (dataflow)
expression

Recursive descent

Σ -OL (loop)
expression

Confluent term rewriting

Optimized Σ -OL
expression

Recursive descent

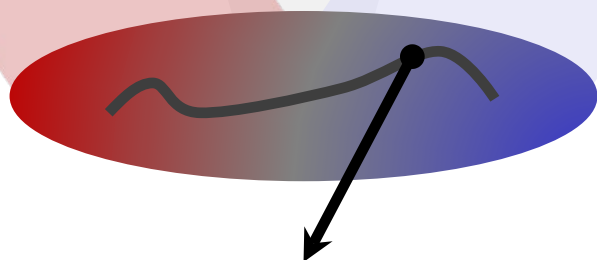
Abstract code

Confluent term rewriting

Optimized abstract
code

Recursive descent

C code



$$\left((L_m^{mp} \otimes I_{n/p\mu}) \otimes I_\mu \right) \left(I_p \otimes_{||} (\text{DFT}_m \otimes I_{n/p}) \right) \left((L_p^{mp} \otimes I_{n/p\mu}) \otimes I_\mu \right) \left(\bigoplus_{i=0}^{p-1} T_n^{mn,i} \right) \left(I_p \otimes_{||} (I_{m/p} \otimes \text{DFT}_n) \right) \left(I_p \otimes_{||} L_{m/p}^{mn/p} \right) \left((L_p^{pn} \otimes I_{m/p\mu}) \otimes I_\mu \right)$$

Translating an OL Expression Into Code

Constraint Solver Input:

$\underbrace{\text{DFT}_8}_{\text{double}}$

Output =

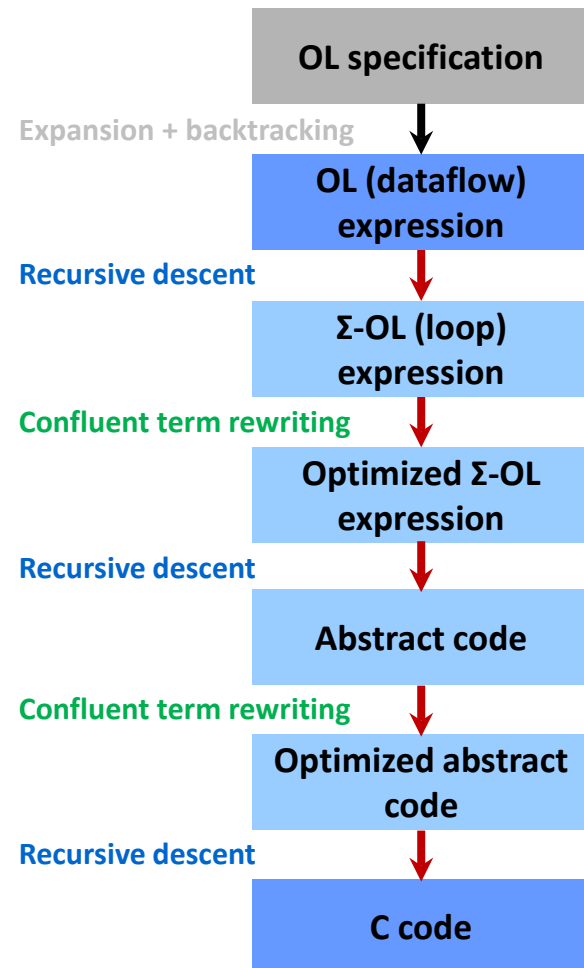
OL Expression: $(\text{DFT}_2 \otimes I_4) T_4^8 (I_2 \otimes ((\text{DFT}_2 \otimes I_2) T_2^4 (I_2 \otimes \text{DFT}_2) L_2^4)) L_2^8$

Σ -OL:

$$\sum_{j=0}^3 (S_j \text{DFT}_2 G_j) \sum_{k=0}^1 \left(\sum_{l=0}^1 (S_{k,l} \text{diag}(t_{k,l}) \text{DFT}_2 G_l) \sum_{m=0}^1 (S_m \text{diag}(t_m) \text{DFT}_2 G_{k,m}) \right)$$

C Code:

```
void sub(double *y, double *x) {
    double f0, f1, f2, f3, f4, f7, f8, f10, f11;
    f0 = x[0] - x[3];
    f1 = x[0] + x[3];
    f2 = x[1] - x[2];
    f3 = x[1] + x[2];
    f4 = f1 - f3;
    y[0] = f1 + f3;
    y[2] = 0.7071067811865476 * f4;
    f7 = 0.9238795325112867 * f0;
    f8 = 0.3826834323650898 * f2;
    y[1] = f7 + f8;
    f10 = 0.3826834323650898 * f0;
    f11 = (-0.9238795325112867) * f2;
    y[3] = f10 + f11;
}
```



STAP in SPIRAL's OL specification

$$(I_D \otimes I_B \otimes (I_V \otimes I_S \otimes (Dot \circ Extract))) \circ (I_V \otimes \frac{1}{\|Dot^{CF}\|_2}) \circ$$

Inner Products

$$(I_D \otimes I_B \otimes I_V \otimes (Trsv_{bwd}^{CF} \circ Trsv_{fwd}^{CF})) \circ$$

Linear System Solver

$$(I_D \otimes I_B \otimes Chol^{CF}) \circ$$

$$(I_B \otimes I_D \otimes (\frac{1}{S} \times (I_S \otimes (Her^{CF} \circ Extract)))) \circ$$

Covariance Estimate

$$(I_C \otimes L_R^{RD}) \circ$$

FFT + Corner Turn

$$(I_C \otimes I_R \otimes DFT_D)$$

STAP: Necessary Optimization

$$(I_D \otimes I_B \otimes (I_V \otimes I_S \otimes (Dot \circ Extract))) \circ (I_V \otimes \frac{1}{\|Dot^{CF}\|_2}) \circ$$

$$(I_D \otimes I_B \otimes I_V \otimes (Trsv_{bwd}^{CF} \circ Trsv_{fwd}^{CF})) \circ$$

$$(I_D \otimes I_B \otimes Chol^{CF}) \circ$$

$$(I_B \otimes I_D \otimes (\frac{1}{S} \times (I_S \otimes (Her^{CF} \circ Extract)))) \circ$$

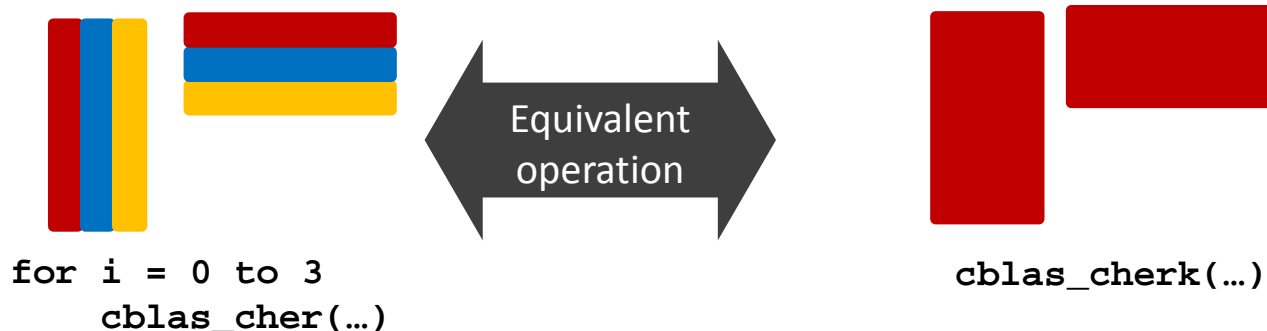
$$(I_C \otimes L_R^{RD}) \circ$$

$$(I_C \otimes I_R \otimes DFT_D)$$

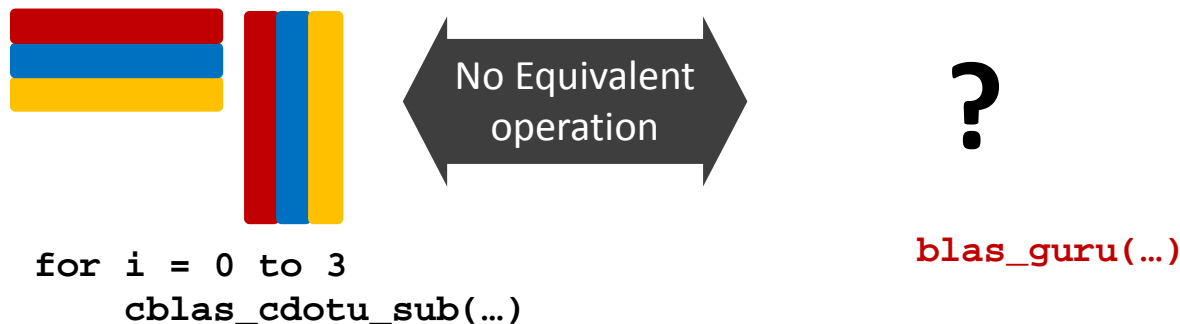
**BLAS2 operations to be
optimized into BLAS3
operations**

Problem: BLAS/LAPACK interface

■ Merged Operation



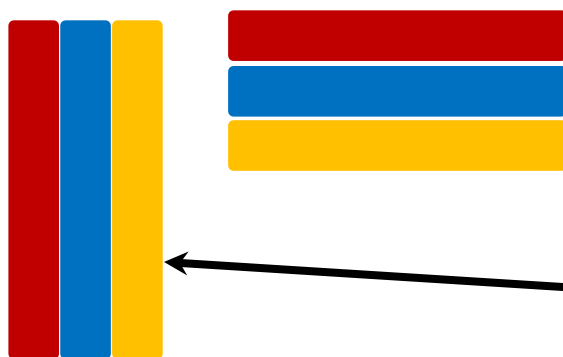
■ Batched Operation



STAP: SPIRAL Optimization

BLAS 2 operation

$$(I_B \otimes I_D \otimes (\frac{1}{S} \times (I_S \otimes (Her^{CF} \circ Extract))))$$



Data stored column-major
instead of row-major as
required

After Spiral's transformations

$$(I_{DB} \otimes (\frac{1}{S} \times (Herk^{CF,S} \circ I_S \otimes (L_{CF}^{SCF} \circ Extract))))$$

Loop interchange +
Loop coalescing

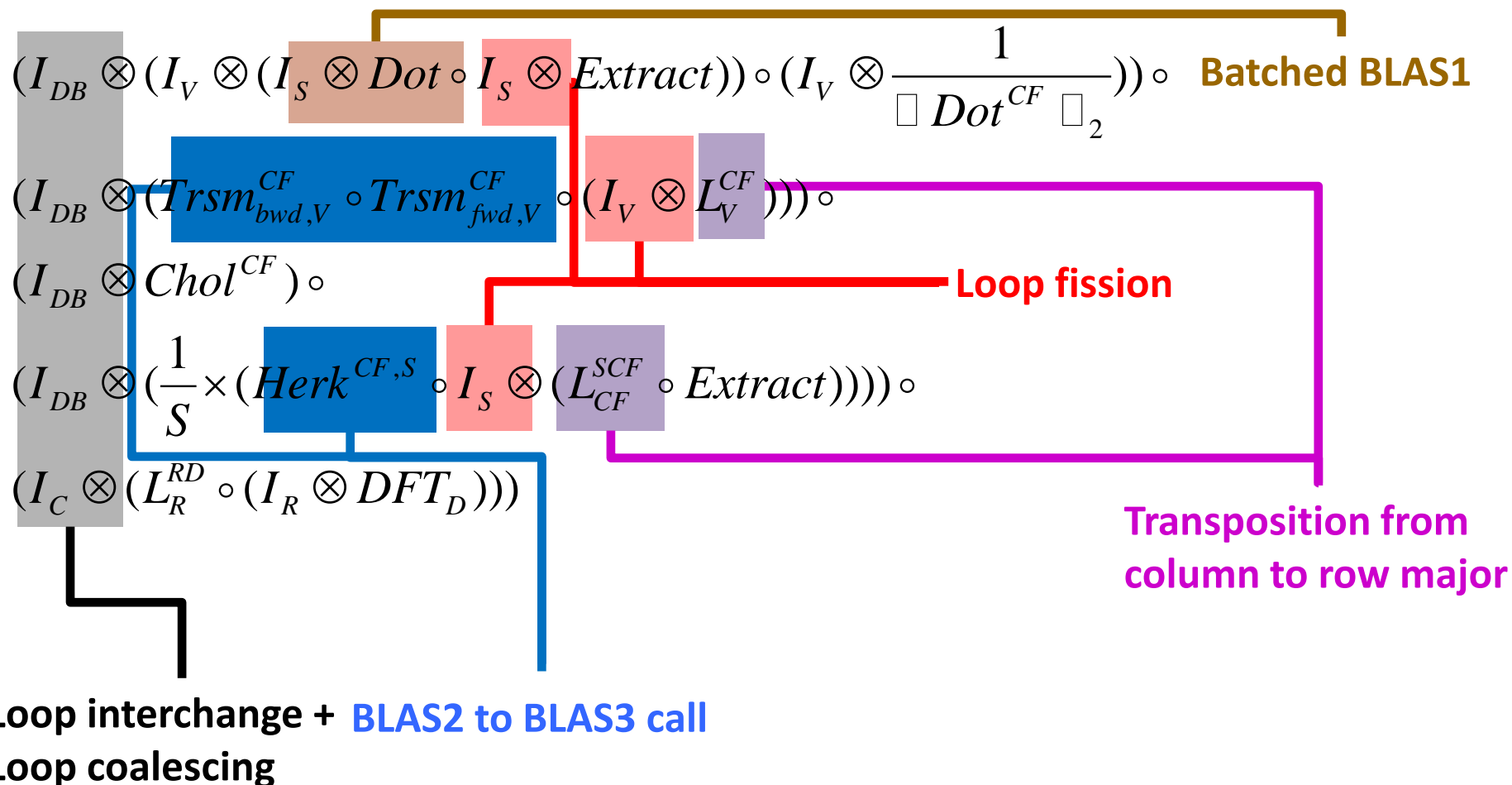
BLAS2 to BLAS3 call

Loop fission

Transposition from
column to row major

Interpreting the OL Specification

After SPIRAL's transformations



OL vs. FFTW Guru Interface

■ FFTW guru-interface

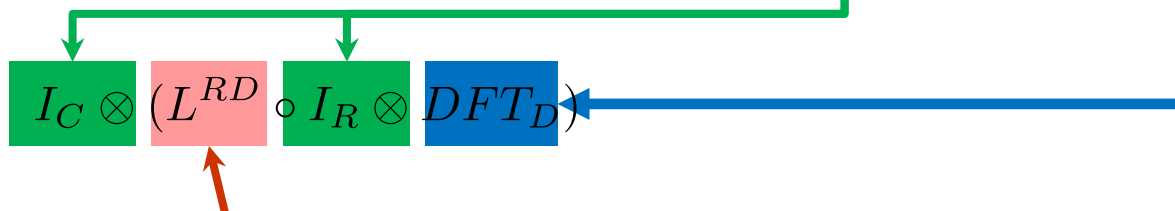
Batched descriptor for FFT computation

```
fftwf_plan_guru_dft(  
    1, dims,  
    2, howmany_dims,  
    datacube_pulse_major_padded,  
    doppler_datacube,  
    FFTW_FORWARD, FFTW_WISDOM_ONLY);
```

Compute descriptor

Batch descriptor

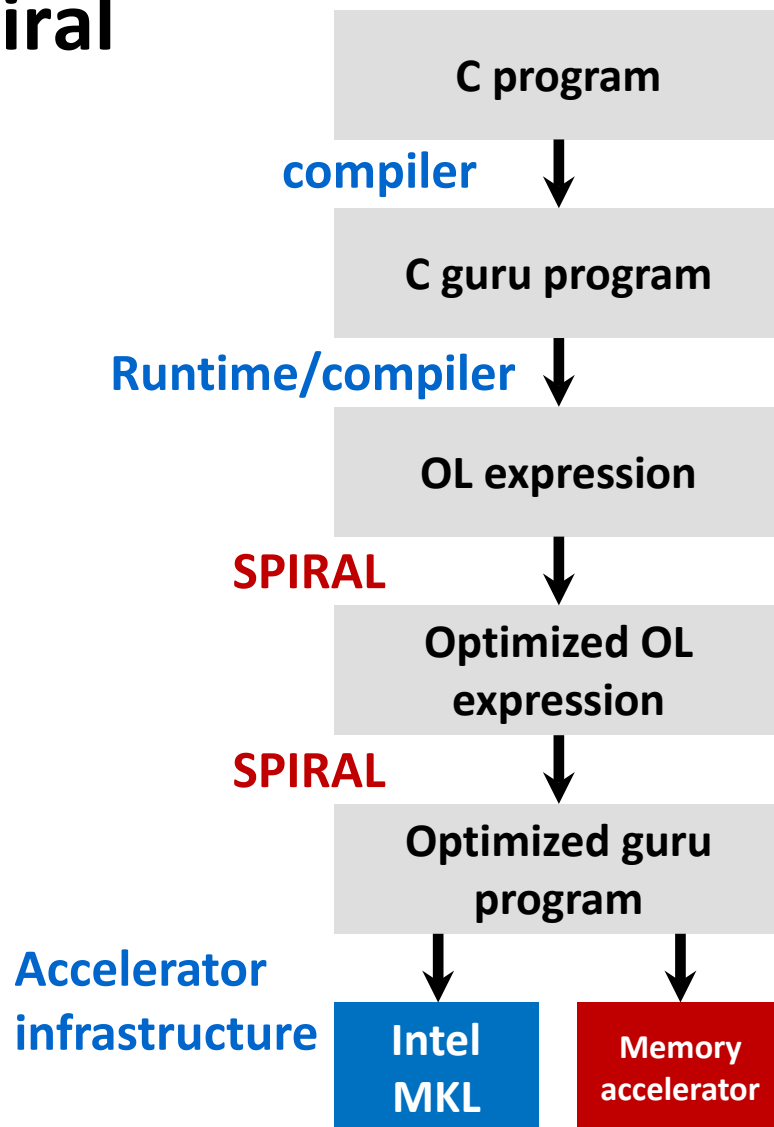
■ OL representation of FFTW guru interface



Implicit Reshape in Compute Descriptor

Optimizing STAP with Spiral

- Input: C program with BLAS and FFTW calls
- **Spiral Input:** STAP C program only using FFTW and BLAS guru calls
- This is equivalent to a OL formula in Spiral
- Enables full-program data layout optimization and kernel merging
- **Final program:** efficient on CPU, can target novel accelerators



This paves the way for our 3DIC memory side accelerator (discussed next)

Outline

- Example: STAP
- Cross-call/cross library optimization with Spiral
- **Library-based hardware acceleration**
- Summary

Overview: Memory-Side Accelerators

■ Accelerator behind DRAM interface

- No off-DIMM data traffic and SERDES
- Huge problem sizes possible
- 3D stacking is enabling technology

■ Configurable array of accelerators

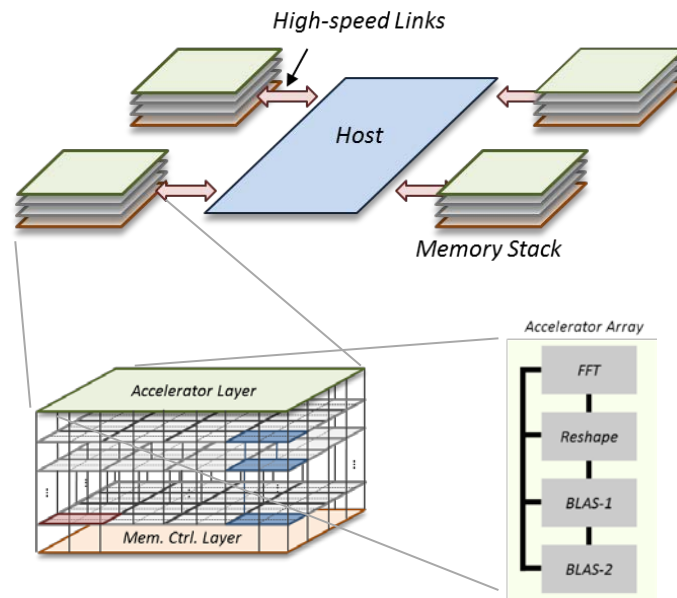
- Domain specific, highly configurable
- Cover DoD-relevant kernels

■ System CPU

- Standard: Multicore CPU+GPU

■ Software stack

- User level: standard numeric libraries BLAS1/2, FFTW, SpMV/SpGEMM,...
- OS, driver, compiler, runtime system support necessary for drop-in replacement



Accelerator Hardware Architecture

■ Accelerator tiles

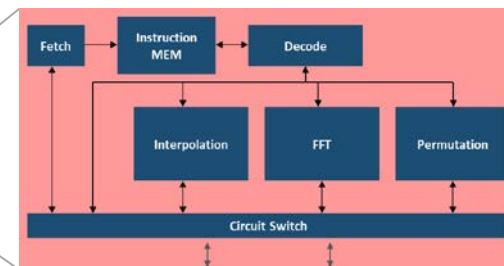
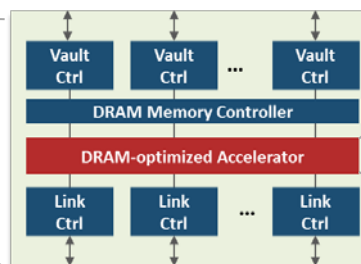
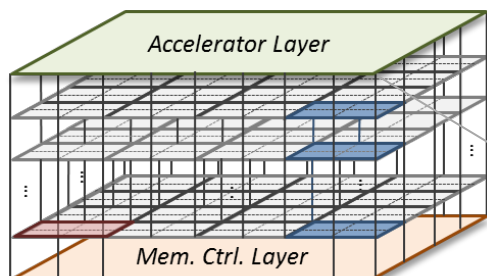
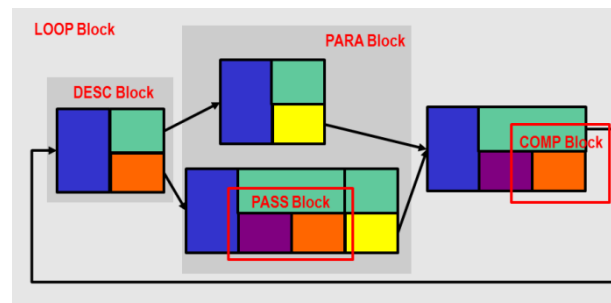
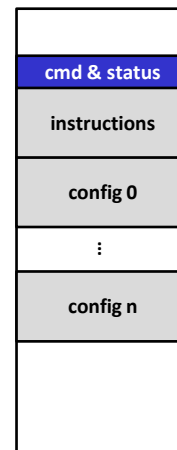
- One accelerator tile per 3D-DRAM vault
- Can be linked to one big accelerator
- Configurable/programmable: units, interconnect, iterators

■ Programming model

- Memory mapped device
- Physical memory addressing
- Data and command segment in DRAM
- Controlled via read/writes to command addresses

```

LOOP
  numLoop = 10;
DESC
  descID = 0;
  numPass = 1;
PASS
  passID = 0;
  numComp = 1;
COMP
  compID = 0;
  Inst = FFT;
  Conf = "fft.conf"
ENDCOMP
ENDPASS
ENDDDESC
...
ENDLOOP
  
```



Accelerator Software Architecture

■ Accelerator software abstraction

- memory mapped device
- command and data address space
- kernel/virtual memory configuration

■ Low level user API

- device driver interface
- low level C configuration library

■ Application level API

- transparent device-aware malloc/free
- standard low intensity math libraries: BLAS-1, BLAS-2, FFTW, sparse mat/vec, copy/reshape, corner turn
- “guru” math library interfaces
- OpenACC directives + proto compiler

```
// TAV STAP example
//
// kernel reimplemented with BLAS and OpenACC
//

float *adaptive_weights;
float *steering_vectors;

#pragma declare \
    device_resident(adaptive_weights);
#pragma declare \
    device_resident(steering_vectors);

adaptive_weights = XMALLOC(sizeof(complex)*
    num_adaptive_weight_elements);
steering_vectors = XMALLOC(sizeof(complex)*
    num_steering_vector_elements);

...
#pragma acc data copyin(adaptive_weights, \
    steering_vectors), copyout(accums)
#pragma acc kernels loop
for (sv = 0; sv < N_STEERING; ++sv)
{
    for (block = 0; block < N_DOP*N_BLOCKS; ++block)
    {
        accum.re = accum.im = 0.0f;
        cblas_cdotc_sub(TDOF*N_CHAN,
            (float*)adaptive_weights[block][sv],
            1,
            (float*)steering_vectors[sv],
            1,
            (float*)&accums[block][sv]);
    }
}
...
```

Hardware in the Loop Simulation

■ Accelerator Simulation

- **Timing:** Synopsis DesignWare
- **Power:** DRAMSim2, Cacti, Cacti-3DD, McPAT

■ Full System Evaluation

- Run code on real system (Haswell, Xeon Phi) or in simulator (SimpleScalar,...)
- Normal DRAM access for CPU, but trap accelerator command memory space, invoke simulator

```
#include "fftw.h"
#include "papi_util.h"
...
{
    fftw_complex *in, *out;
    fftw_plan p;
    ...
    in = (fftw_complex*)
        fftw_malloc(sizeof(fftw_complex) * N * N);

    out = (fftw_complex*)
        fftw_malloc(sizeof(fftw_complex) * N * N);

    p = fftw_plan_dft_2d(N, N, in, out,
        FFTW_FORWARD, FFTW_MEASURE);

    ///// Measurement Code /////
    papi_begin();
    //////////////////////////////////

    fftw_execute(p);

    ///// Measurement Code /////
    papi_end();
    //////////////////////////////////

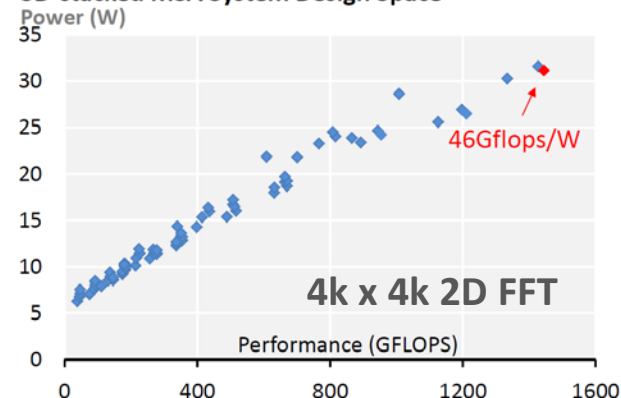
    fftw_destroy_plan(p);
    fftw_free(in); fftw_free(out);
}
```

Synopsis
DesignWare

McPAT



3D-stacked MSA System Design Space



Accelerating the TAV STAP Benchmark

■ STAP structure: 4 phases

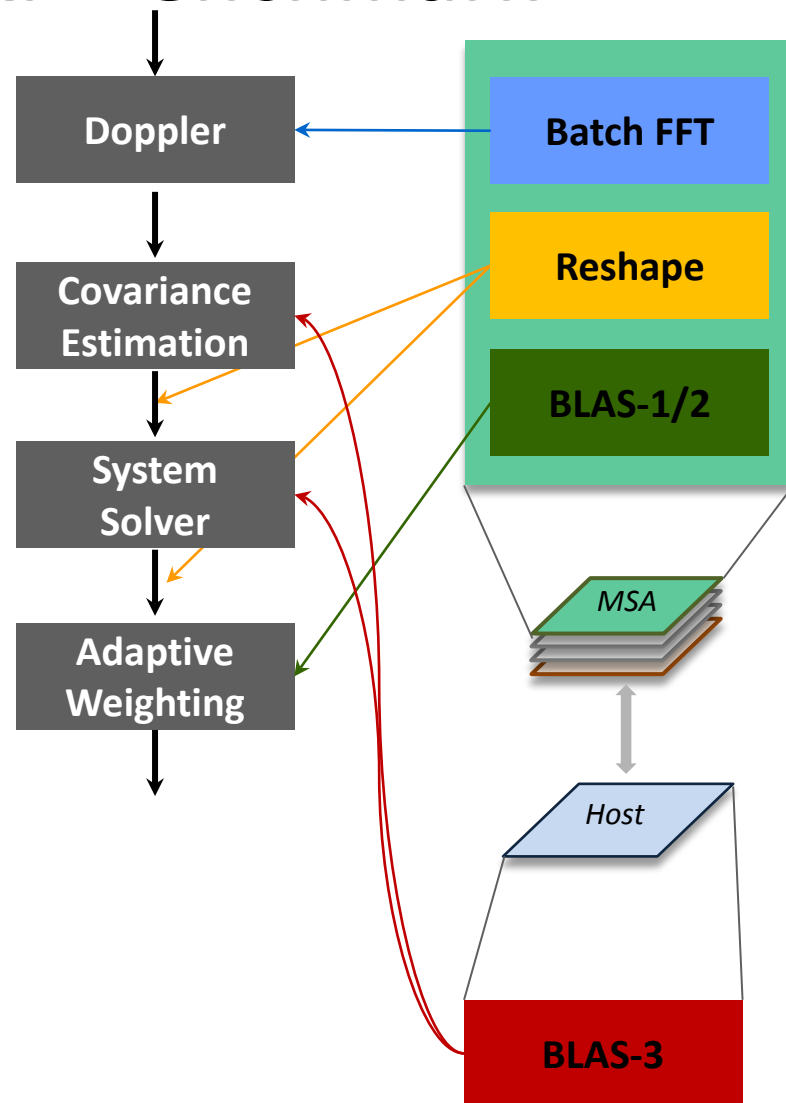
- Doppler: batch FFT
- Covariance estimation: outer product
- System solver: Cholesky or QR
- Adaptive weighting: inner product

■ STAP memory-side acceleration

- On CPU: BLAS-3/Cholesky
- On accelerator: FFT, BLAS-1, BLAS-2

■ Software interface

- Drop-in replacement for Intel MKL: FFTW, BLAS-1/2/3
- OpenACC pragmas for batch BLAS-1/2

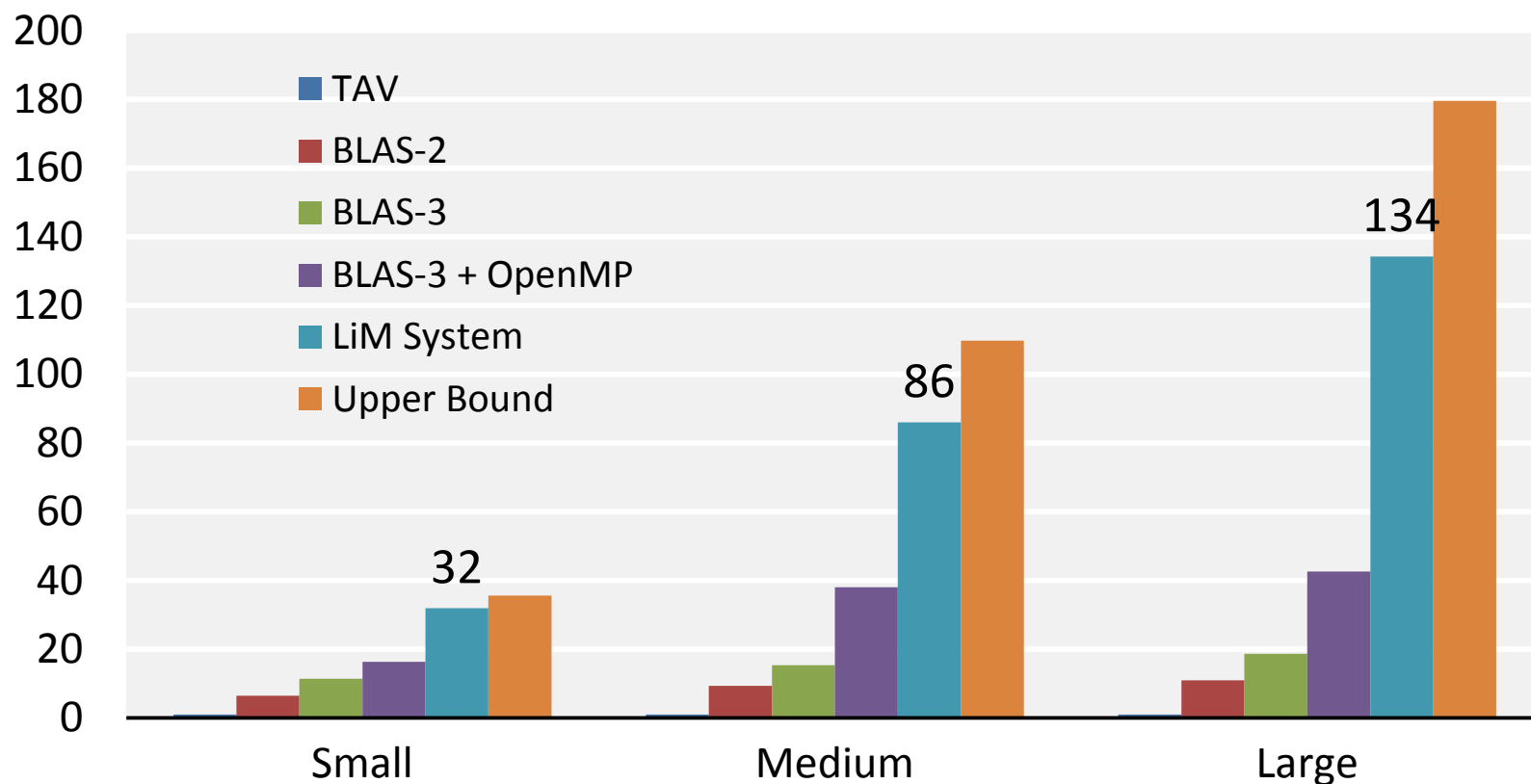


No DMA/copy-in/copy-out necessary for accelerator

STAP Performance Results

Performance Gains over Baseline (TAV)

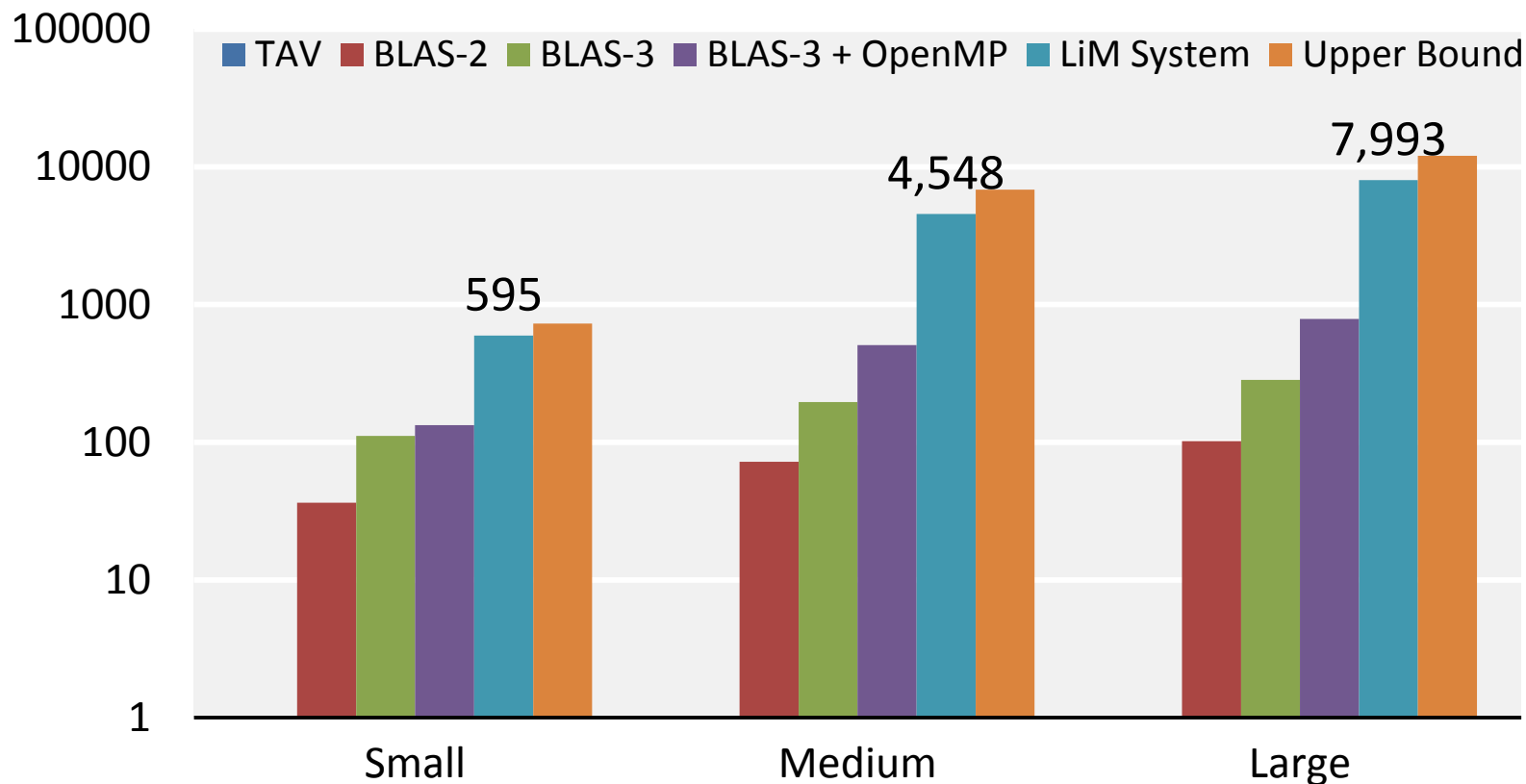
Speedup (Times)



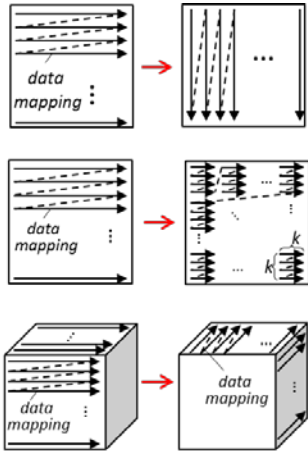
STAP Power Efficiency Results

GFLOPS/W Gain over Baseline (TAV)

EDP Reduction (Times)

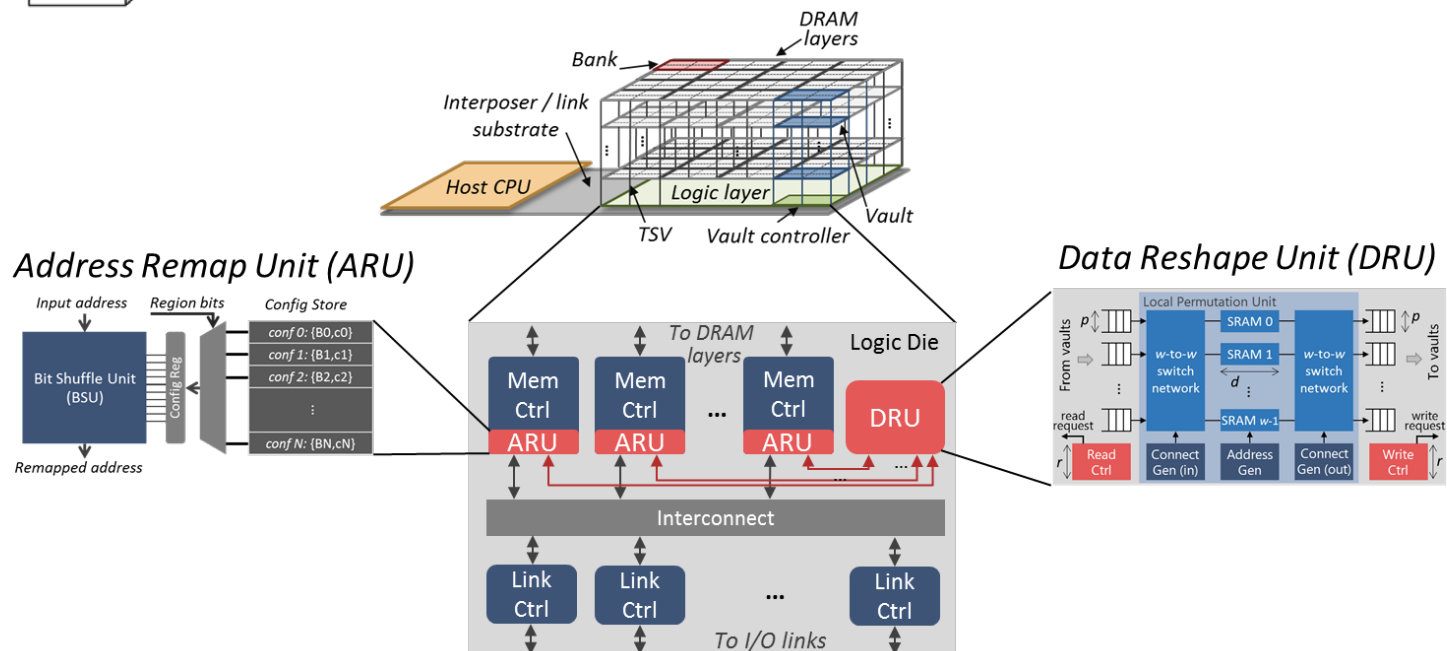


Memory Layout Accelerator



Intel MKL (Math Kernel Library) functions:

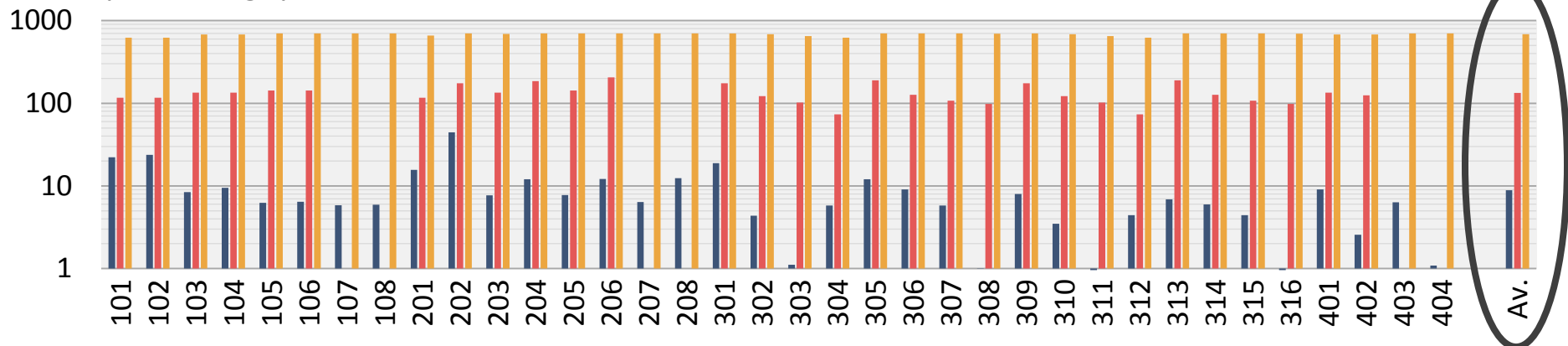
```
void mkl_simatcopy (const char ordering, char trans, size_t rows, size_t cols,
                    const float alpha, float * AB, size_t lda, size_t ldb);
void mkl_somatcopy (char ordering, char trans, size_t rows, size_t cols,
                    const float alpha, float * A, size_t lda, float * B, size_t ldb);
void pstrmr2d (char *uplo, char *diag, MKL_INT *m, MKL_INT *n, float *a,
              ..., MKL_INT *jb, MKL_INT *descb, MKL_INT*ictxt ); // ScaLAPACK
vsPackI ( n, a, inca, y );
vsUnpackI ( n, a, y, incy );
void cblas_scopy (const MKL_INT n, const float *x, const MKL_INT incx, ...);
void cblas_sswap (const MKL_INT n, float *x, const MKL_INT incx, float *y,...);
pslared2d (n, ia, ja, desc, byrow, byall, work, lwork)
pslaswp (direc, rowcol, n, a, ia, ja, desca, k1, k2, ipiv)
...
```



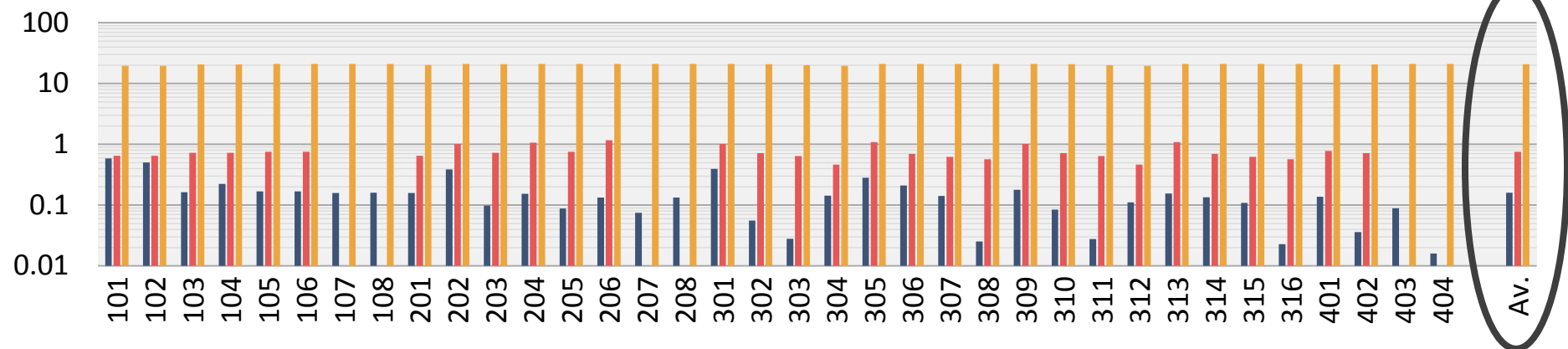
Accelerating MKL Reorganization Routines

■ CPU (i7-4770K) ■ GPU (GTX 780) ■ DRU (MH)

Reshape Throughput [GB/s]



Energy Efficiency [GB/J]

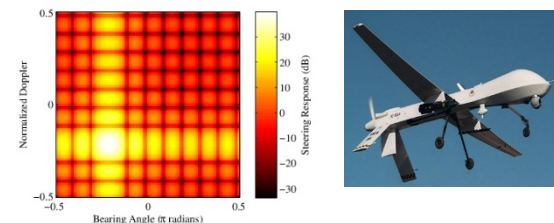


Outline

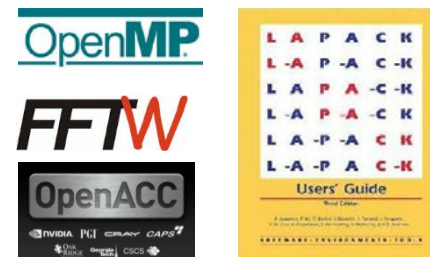
- Example: STAP
- Cross-call/cross library optimization with Spiral
- Library-based hardware acceleration
- Summary

Summary

- Target: dense scientific and HPEC kernels
math heavy, regular, good library coverage



- HPC Library + OpenMP + C as DSL
View program as specification



- Enormous efficiency gains are possible
Novel accelerators + Spiral optimization

